Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Handout 25
April 27,1995

Problem Set 9

Due: In recitation, May 5.

Problem 9-1. [10 points] Suppose A, B, C are independent events. Show that any of the combinations

$$A^c, B, C;\ A, B^c, C; \cdots; A^c, B^c, C; \cdots; A^c, B^c, C^c$$

are also independent. Furthermore, show that A and $B \cup C$ are independent.

Problem 9-2. [10 points] Chuck-A-Luck is played in many casinos. One version of it is as follows. There is a cage with three dice in it. First the player bets \$1 on a number between 1 and 6. The three dice are then rolled. If i dice ($i = 1, 2$ or 3) show his number, he wins \$$i$ plus getting his initial \$1 bet back; if none of the dice shows the number, he loses his bet.

Define the expected value of some random variable X to be $E(X) = \sum_{\text{all } x} x \cdot \text{Prob}\{X = x\}$.

For a player starting with \$1, what is his expected wealth after one play? (That is, the sum over all outcomes of the probability of that outcome times the wealth he ends up with given that outcome.)

Problem 9-3. [10 points]

You are given a biased coin and are told that it could either have $\frac{1}{3}$ probability getting heads and $\frac{2}{3}$ tails or the other way around. Your best knowledge is that it is equal likely to be either kind. So you flip the coin 12 times and you get 7 heads and 5 tails. What is your best guess about the type of coin and what is your estimate of the probability that you are right?

Problem 9-4. [10 points] Once upon a time, a soldier was sentenced to death. To show his kindness, the king decided to give him another chance. He gave the soldier 50 black balls and 50 white balls and 2 pots and told him to put the balls in the two pots whichever way he preferred. Then the king would come back and randomly take away one pot and mix the other. The soldier then pick a ball from the remaining pot and live if and only if it was white. What strategy should the soldier take? Prove that it is the best.

Problem 9-5. [10 points] Consider in a country where people favor boys to girls and each family will keep having babies until they have a boy. (And then they stop having kids.) Assume that genetically boys and girls are equally likely to be born, and that the death rate is the same for both sexes. Will there be a bias in that country's population in the long run? Explain your answer.

Problem 9-6. [10 points] Mr. James W. Pitman received a letter:
" $\cdots$ to inform you that one of the following three people has won a New Datsun Sentra: Collin Andrus; James W. Pitman; Larry Abbott. $\cdots$"
" $\cdots$ you will receive one of the following for sure:
(1) RCA Color TV; (2) 5 FT. Grandfather Clock; (3) Datsun Nissan sentra.
To claim your gift, call $\cdots$"
According to small print on the back of the letter:
" $\cdots$ the retail values and chance of receiving each gift are No. 1– $1/10,000$ ($329.95), No.2– $9998/10,000$ ($249.95), No.3– $1/10,000$ ($5,995.00). $\cdots$ "
Let us assume that this is an honestly conducted sweepstake, and each of the three individuals named above had originally a 1 in $10,000$ chance of winning the new Datsun. Now that Mr. Pitman know that the winner is one of the three individuals including him, the rules of conditional probability imply that he has a $\frac{1}{3}$ chance of winning the Datsun. True or False? Explain!