Quiz 2, Conflict Exam

- This quiz is open book.
- Put your name on the top of each page—the pages of this booklet will be separated for grading.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please do not put part of the answer to one problem on the back of the sheet for another problem.
- Do not spend too much time on any problem. Read them all through first and attack them in the order that allows you to make the most progress.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. **Be neat**.
- This is a two-hour quiz. It contains six problems. The problems are 16 points each, and you will get 4 points for writing your name, tutorial TA and time clearly.
- GOOD LUCK!

Problem	Points	Grade
name	4	
1	16	
2	16	
3	16	
4	16	
5	16	
6	16	
Total	100	

Name: ____________________

Please circle your tutorial TA: Jennifer Parry Steve Ravi

Please circle your tutorial time: 10 11 12 1 2 3

Problem 1. [16 points]

Let the universe be the nodes V of a graph. Let $E(x,y) = 1$ iff there is an edge from node x to node y. Let $A : V \mapsto \{0,1\}$ be a function from V to $\{0,1\}$. R, B, G are also functions from V to $\{0,1\}$. What graph properties do the following sentences express?

(a) $\exists A\,[\exists x,y\,(A(x) \wedge \neg A(y))] \wedge [\forall x,y\,((A(x) \wedge \neg A(y)) \rightarrow \neg E(x,y))]$

(b) $\exists R, B, G,$

$$
\begin{array}{llllllll}
[& (\forall x & (R(x) & \wedge & \neg B(x) & \wedge & \neg G(x)) & \vee \\
 & & (\neg R(x) & \wedge & B(x) & \wedge & \neg G(x)) & \vee \\
 & & (\neg R(x) & \wedge & \neg B(x) & \wedge & G(x)) &) \\
\wedge & (\forall x,y & \multicolumn{5}{l}{((R(x) \wedge R(y)) \rightarrow \neg E(x,y))} & \wedge \\
 & & \multicolumn{5}{l}{((B(x) \wedge B(y)) \rightarrow \neg E(x,y))} & \wedge \\
 & & \multicolumn{5}{l}{((G(x) \wedge G(y)) \rightarrow \neg E(x,y))} &) \quad]
\end{array}
$$

Problem 2. [16 points]

Prove of disprove: $f(n) = \Theta(f(n/2))$.

Problem 3. [16 points]

Solve the following recurrence:

$$f(1) = f(2) = 1$$

$$f(n) + f(n-2) = 2^n$$

Problem 4. [16 points]

Give asymptotic upper and lower bounds for $T(n)$ (i.e. $T(n) = \Theta(f(n))$). Assume that $T(n)$ is constant for $n \leq 2$.

(a) $T(n) = 6T(n/3) + n^2$

(b) $T(n) = 3T(n/9) + \sqrt{n}$

(c) $T(n) = 5T(n/3) + n$

Problem 5. [16 points]

Let $c(n, k)$ be the number of permutations of $\{1, \cdots, n\}$ with exactly k cycles. State and prove a recurrence for $c(n, k)$ in terms of $c(n-1, k)$ and $c(n-1, k-1)$.

Problem 6. [16 points]

How many graphs are there on 5 nodes that has no vertex of degree 4?