Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Handout 18
April 6,1995

Problem Set 7

Due: In recitation, April 14.

Problem 7-1. Solve the following recurrences:

(a) $f(n) = 2f(n-1) - 2f(n-2);\ f(0) = 0,\ f(1) = 1.$

(b) $f(n) = 7f(n-1) - 16f(n-2) + 12f(n-3);\ f(0) = 0,\ f(1) = 1,\ f(2) = 2.$

(c) $f(n) = 2f(n-1) + n;\ f(0) = 1.$

Problem 7-2. Prove that the Vandermonde matrix $\begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ \alpha_1^1 & \alpha_2^1 & \alpha_3^1 & \cdots & \alpha_d^1 \\ \vdots & & \vdots & \ddots & \\ \alpha_1^{d-1} & \alpha_2^{d-1} & \alpha_3^{d-1} & \cdots & \alpha_d^{d-1} \end{bmatrix}$ has determinant $\prod_{0 \le j < k \le d-1} (\alpha_k - \alpha_j)$.

Problem 7-3. **(a)** How many ways can $2n$ people be divided into n pairs?

(b) How many ways can you choose n out of $2n$ objects, given that n of the $2n$ objects are identical?

Problem 7-4. A pizza house is having a promotional sale. Their commercial reads: "... buy 2 large pizzas at regular price, get up to 11 toppings for each pizza absolutely free. That's 4194304 different ways to design your order!!!! ..."

(a) Show that $4194304 = \left(\sum_{k=0}^{11} \binom{11}{k} \right)^2$ is actually wrong and give the right answer for the number of ways that you can choose toppings for the two pizzas.

(b) Now assume that you can choose any number of the same topping as long as the total number of toppings is no more than 11. (That is, you can have 2 meatball, 4 ham and 5 sausage toppings if you are truly a meat lover.) How many different ways can you now choose toppings for your two pizzas?