Massachusetts Institute of Technology — Quiz 1
6.042J/18.062J: Mathematics for Computer Science — March 15, 1995
Professors Mauricio Karchmer and Ron Rivest

Quiz 1

- This quiz is open book.
- Put your name on the top of each page—the pages of this booklet will be separated for grading.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please do not put part of the answer to one problem on the back of the sheet for another problem.
- Do not spend too much time on any problem. Read them all through first and attack them in the order that allows you to make the most progress.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. **Be neat.**
- This quiz ends at 9 P.M. It contains six problems. The problems are 16 points each, and you will get 4 points for writing your name, recitation TA and time clearly.
- GOOD LUCK!

Problem	Points	Grade
name	4	
1	16	
2	16	
3	16	
4	16	
5	16	
6	16	
Total	100	

Name: ______________________________

Please circle your recitation TA: Ravi Steve

Please circle your recitation time: 1 2 3

Problem 1. [16 points]

Discover and prove a formula for $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n$.

Problem 2. [16 points]

Prove that the following two sequences are equal:

$$\begin{aligned} u_0 &= 2 \\ u_{n+1} &= u_n^2 - u_n + 1 \end{aligned}$$

$$\begin{aligned} v_0 &= 2 \\ v_{n+1} &= 1 + v_0 v_1 \cdots v_n \end{aligned}$$

Problem 3. [16 points]

For a function $f : A \mapsto B$ and a subset $C \subseteq A$, define $f(C) = \{b \mid f(a) = b \text{ for some } a \in C\}$. Prove that if f is a 1-1(an injection), then $f(C \cap D) = f(C) \cap f(D)$ for every $C, D \subseteq A$.

Problem 4. [16 points]

Define the poset $(\{1, 2, 3, 4, 5\}^2, \prec)$ where $(a, b) \prec (c, d)$ iff $a \leq c$ and $b \leq d$.

(a) Give an antichain which is as long as possible.

(b) Prove that the above antichain is the largest.

Problem 5. [16 points] The *complement* of an undirected graph $G = (V, E)$ is the graph $G^c = (V, E^c)$. That is, G^c contains exactly those edges that are not in G. Prove that if G is not connected then G^c is.

Problem 6. [16 points]

Give an explicit bijection between $\mathbb{R}$ and $(0, \infty)$, where $(0, \infty)$ is the set $\{x \in \mathbb{R} \mid x > 0\}$.