Problem Set 3

Due: In recitation, March 3, 1995.

Problem 3-1. Let n be a natural number greater than 2. Define relation R on $\mathbb{Z}$ by

$$a \, R \, b \iff n \text{ divides } a - b.$$

(a) Prove that R is an equivalence relation.

(b) Identify the total number of equivalence classes and describe them.

Problem 3-2. The following relation $\prec$ defines a partial order on $\{2, 3, 4, \cdots, 100\}$:

$$a \prec b \iff a \text{ divides } b.$$

(a) Describe the minimal and maximal elements of this poset.

(b) Find a longest chain.

(c) Find a smallest antichain cover and prove that it is the smallest.

Problem 3-3. Prove that any partial ordering of a finite set can be extended to a total ordering of that set.

Problem 3-4. Let $\prec$ be a partial ordering. Define R by

$$b \, R \, a \iff a \prec b.$$

Show that R is also a partial ordering.