Massachusetts Institute of Technology — Handout 5
6.042J/18.062J: Mathematics for Computer Science — February 16, 1995
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Problem Set 2

Due: In recitation, February 24.

Problem 2-1. Define the composition $P \circ Q$ of two relations as follows:
If $Q \subseteq A \times B$ and $P \subseteq B \times C$ then $P \circ Q = \{(a, c) : \exists b \text{ such that } (a, b) \in Q \text{ and } (b, c) \in P\}$. Note that $P \circ Q \subseteq A \times C$.

Let $R \subseteq S \times S$ be a relation. Prove or disprove each of the following statements.

(a) If R is transitive, then $R \circ R$ is transitive.

(b) If R is transitive, then $R \circ R \subseteq R$.

(c) If $R \circ R \subseteq R$, then R is transitive.

Problem 2-2. Prove or disprove the following identities:

(a) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(b) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times D)$

(c) $(A - B) \times (C - D) = (A \times C) - (B \times D)$

(d) $(A \bigtriangleup B) \times (C \bigtriangleup D) = (A \times C) \bigtriangleup (B \times D)$

Problem 2-3. Let $f^{(i)}(x)$ denote the ith derivative of $f(x)$. Prove that if $f(x)$ and $g(x)$ are real-valued functions of one variable with derivatives of all orders, then

$$(f(x)g(x))^{(n)} = \sum_{j=0}^{n} \binom{n}{j} f^{(n-j)}(x) g^{(j)}(x)$$

Problem 2-4. You are given an unlimited supply of 4-cent and 7-cent stamps. What is the largest amount of postage that you cannot make with these two denominations? Prove your answer.