Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Handout 3
February 9, 1995

Problem Set 1

Due: In recitation, February 17.

Reading: Chapter 1 of *Discrete Mathematics for Computer Scientists* by J.K. Truss.

Problem 1-1. Let F_i be the *ith* Fibonacci number; that is,

$$F_0 = 0, \ F_1 = 1, \quad \text{and} \quad F_i = F_{i-1} + F_{i-2}, \quad \text{for } i \geq 2.$$

Show that $1 + \sum_{i=0}^{k} F_i = F_{k+2}$ for all $k \geq 0$.

Problem 1-2. Prove that for each integer $n > 1$, the expression

$$\underbrace{\sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}}_{n \text{ 1's}}$$

is irrational. For example, for $n = 2$ this is $\sqrt{1 + \sqrt{1}}$. You may assume that $\sqrt{2}$ is irrational.

Problem 1-3. Define the binomial coefficient $\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$ for $0 \leq k \leq n$. This is read "n choose k," and is the number of ways of choosing k distinct objects from a set of n distinct objects. (Although you can't use this fact in your proofs unless you prove it first.) Don't forget that 0! is defined to be 1. The binomial coefficient is defined to be 0 if $k < 0$ or $k > n$.

Show that

(a) $$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1} \quad \text{for any integers } n \text{ and } k \text{ such that } 0 \leq k \leq n,$$

and

(b) $$\sum_{k=0}^{n} \binom{k}{m} = \binom{n+1}{m+1} \quad \text{for any nonnegative integers } n \text{ and } m.$$

Problem 1-4. Let θ be an angle for which $\sin\theta$ and $\cos\theta$ are rational. Prove that for all integers n, $\sin n\theta$ and $\cos n\theta$ are rational.

Problem 1-5. Show that 5 divides $3^{2k} + 4^{k+1}$ for all integers $k \geq 0$.