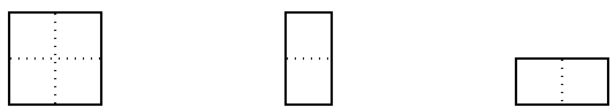
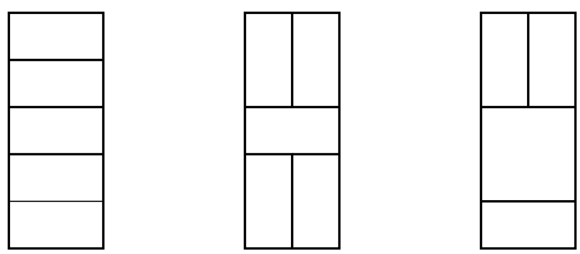
Problems for Recitation 4

1 Problem: Mini-Tetris

A *winning configuration* in the game of Mini-Tetris is a complete tiling of a $2 \times n$ board using only the three shapes shown below:



For example, here are several possible winning configurations on a 2×5 board:



1. Let T_n denote the number of different winning configurations on a $2 \times n$ board. Determine the values of T_1, T_2, and T_3.

2. Express T_n in terms of T_{n-1} and T_{n-2}.

3. Using strong induction, prove that the number of winning configurations on a $2 \times n$ Mini-Tetris board ($n \geq 1$) is:

$$T_n = \frac{2^{n+1} + (-1)^n}{3}$$

2 Problem: Breaking a chocolate bar

We are given a chocolate bar with $m \times n$ squares of chocolate, and our task is to divide it into mn individual squares. We are only allowed to split one piece of chocolate at a time using a vertical or a horizontal break.

For example, suppose that the chocolate bar is 2×2. The first split makes two pieces, both 2×1. Each of these pieces requires one more split to form single squares. This gives a total of three splits.

Use strong induction to conclude the following:

Theorem. *To divide up a chocolate bar with $m \times n$ squares, we need at most $mn - 1$ splits.*

3 Problem: Fibonacci numbers

The Fibonacci numbers are defined as follows:

$F_1 = 1, F_2 = 1$, and for all $k \geq 3$, $F_k = F_{k-1} + F_{k-2}$.

The first few terms of the Fibonacci sequence are:

$$1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

We can't find every single number in the Fibonacci sequence – for instance, 4 is not a number in the sequence. But can we express every $n \geq 1$ as the sum of distinct terms in the Fibonacci sequence? Indeed, we can!

Use strong induction to prove the following:

Theorem 1. *Every $n \geq 1$ can be expressed as the sum of distinct terms in the Fibonacci sequence.*