

Problems for Recitation 22

Problem 1. Final exams in 6.042 are graded according to a rigorous procedure:

- With probability $\frac{4}{7}$ the exam is graded by a *recitation instructor*, with probability $\frac{2}{7}$ it is graded by a *lecturer*, and with probability $\frac{1}{7}$, it is accidentally dropped behind the radiator and arbitrarily given a score of 84.
- *Recitation instructors* score an exam by scoring each problem individually and then taking the sum.
 - There are ten true/false questions worth 2 points each. For each, full credit is given with probability $\frac{3}{4}$, and no credit is given with probability $\frac{1}{4}$.
 - There are four questions worth 15 points each. For each, the score is determined by rolling two fair dice, summing the results, and adding 3.
 - The single 20 point question is awarded either 12 or 18 points with equal probability.
- *Lecturers* score an exam by rolling a fair die twice, multiplying the results, and then adding a “general impression” score.
 - With probability $\frac{4}{10}$, the general impression score is 40.
 - With probability $\frac{3}{10}$, the general impression score is 50.
 - With probability $\frac{3}{10}$, the general impression score is 60.

Assume all random choices during the grading process are mutually independent.

- (a) What is the expected score on an exam graded by a recitation instructor?

(b) What is the expected score on an exam graded by a lecturer?

(c) What is the expected score on a 6.042 exam?

Problem 2. Here's yet another fun 6.042 game! You pick a number between 1 and 6. Then you roll three fair, independent dice.

- If your number never comes up, then you lose a dollar.
- If your number comes up once, then you win a dollar.
- If your number comes up twice, then you win two dollars.
- If your number comes up three times, you win *four* dollars!

What is your expected payoff? Is playing this game likely to be profitable for you or not?

Problem 3. The number of squares that a piece advances in one turn of the game Monopoly is determined as follows:

- Roll two dice, take the sum of the numbers that come up, and advance that number of squares.
 - If you roll *doubles* (that is, the same number comes up on both dice), then you roll a second time, take the sum, and advance that number of additional squares.
 - If you roll doubles a second time, then you roll a third time, take the sum, and advance that number of additional squares.
 - However, as a special case, if you roll doubles a third time, then you go to jail. Regard this as advancing zero squares overall for the turn.
- (a) What is the expected sum of two dice, given that the same number comes up on both?
- (b) What is the expected sum of two dice, given that different numbers come up? (Use your previous answer and the Total Expectation Theorem.)

- (c) To simplify the analysis, suppose that we always roll the dice three times, but may ignore the second or third rolls if we didn't previously get doubles. Let the random variable X_i be the sum of the dice on the i -th roll, and let E_i be the event that the i -th roll is doubles. Write the expected number of squares a piece advances in these terms.
- (d) What is the expected number of squares that a piece advances in Monopoly?