Prove: $(\exists x)(P(x) \vee Q(x)) \implies (\exists x)P(x) \wedge (\exists x)Q(x)$.

1. Assume $(\exists x)(P(x) \vee Q(x))$	
2. Fix x such that $P(x) \vee Q(x)$	EI
3. $(\exists x)P(x) \wedge (\exists x)Q(x)$	
if $P(x)$ then $(\exists x)P(x)$	EG
if $Q(x)$ then $(\exists x)Q(x)$	EG
QED	cases
4. QED	

Prove: $\forall n \in N(n^2 + n \text{ is even})$.

1. Fix $n \in N$.
2. $n^2 + n$ is even.
    1. If n is odd then $n^2 + n$ is even. ???
    2. If n is even then $n^2 + n$ is even. ???
    3. QED Cases
3. QED

1. If n is odd then $n^2 + n$ is even.
    1. Assume n is odd.
    2. $n^2 + n$ is even.
        1. $\exists x \in N(n = 2x + 1)$ — Def. of "odd"
        2. Choose $x \in N$, $n = 2x + 1$ — Existential Instantiation (EI) (If there *exists* one, you can *choose* one.)
        3. $n^2 + n = 4x^2 + 4x + 1 + 2x + 1 = 2(2x^2 + 3x + 1)$ — Algebra
        4. $\exists y \in N(n^2 + n = 2y)$ — Existential Generalization (EG) (If you *have* one, then one *exists*.)
        5. QED — Def. of "even"
    3. QED — Implication

2. If n is even then $n^2 + n$ is even.
    1. Assume n is even.
    2. $n^2 + n$ is even.
        1. $\exists x \in N(n = 2x)$ Def. of "even"
        2. Choose $x \in N$, $n = 2x$ EI
        3. $n^2 + n = 4x^2 + 2x = 2(2x^2 + x)$ Algebra
        4. $\exists y \in N(n^2 + n = 2y)$ EG
        5. QED Def. of "even".
    3. QED Implication

2. If n is even then $n^2 + n$ is even.
    1. Assume n is even.
    2. Choose $x \in N$, $n = 2x$ — Def. of "even", EI
    3. $n^2 + n = 4x^2 + 2x = 2(2x^2 + x)$ — Algebra
    4. $n^2 + n$ is even. — EG, def. of "even"
    5. QED — Implication

Prove: $\sqrt{2}$ is irrational.

1. $\sqrt{2}$ rational $\Rightarrow false$
    1. Assume $\sqrt{2}$ rational.
    2. $false$
        1. Choose $a, b \in N^+$, $\sqrt{2} = \frac{a}{b}$, lowest terms. — Basic properties of rationals, EI
        2. $2b^2 = a^2$ — Algebra
        3. a^2 is even. — EG, definition of "even"
        4. a is even. — ???
        5. Choose $c \in N$, $a = 2c$. — Definition of "even", EI
        6. $4c^2 = a^2 = 2b^2$ — Algebra
        7. $2c^2 = b^2$ — Algebra
        8. b^2 is even. — EG, definition of "even"
        9. b is even. — ???
        10. a and b aren't in lowest terms. — 1.2.4, 1.2.9
        11. QED — 1.2.1, 1.2.10, propositional logic
    3. QED — Implication.
2. QED — Contradiction.

Prove: $(\forall x)P(x) \implies Q(x)$	
1. $(\forall x)\neg Q(x) \implies \neg P(x)$	
1. Choose x	UI
2. Assume $\neg Q(x)$	
3. $\neg P(x)$	
1. $\neg Q(x)$ means x odd	Def $Q(x)$
2. so x^2 odd	As in birthday proof
3. which means $\neg P(x)$	definition of P
4. QED	UG
4. QED	Direct proof
2. QED	logical tautology

Assume: $x \in N$, x^2 is even.
Prove: x is even.

1. x is odd $\Rightarrow false$	
1. Assume x is odd.	
2. x^2 is odd.	proof like in birthdate example.
3. $false$	Assumption, 1.2.4, logic
2. QED	Contradiction