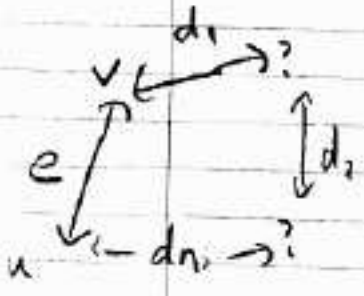
1. Prove that an edge e in a connected graph G is a cut edge if and only if no simple cycle contains e

Prove technique = Contradiction

1. If edge e in G is a cut edge, then there will be no simple cycle containing e
   1. Assume e is a cut edge and there is a simple cycle containing e.
   2. ∃ path from u to v where u, v ∈ G (G is connected)
   3. If we remove e, 1, 2 still true
      1. Let $d_1, d_2, d_3, \ldots d_n$ be the edges in a simple cycle, then there is a path just like that:
         $e \rightleftarrows d_1 \rightleftarrows d_2 \rightleftarrows d_3 \ldots \rightleftarrows d_n \rightleftarrows e$,
      2. Pick arbitary edge e, which connects u to v
      3. Removing it, there is still a path from u to v, say: $d_1 \rightarrow d_2 \rightarrow d_3 \rightarrow \ldots d_n$
      4. Therefore we can construct a path from u to v in the absence of e

      QED
   4. This contradicts the definition of cut edge
      1. Since e is removed, the Graph is disconnected (∵ of cut edge)
      2. There shouldn't exist a path from arbitary u, v if the graph is disconnected
      3. Contradiction

      QED

QED