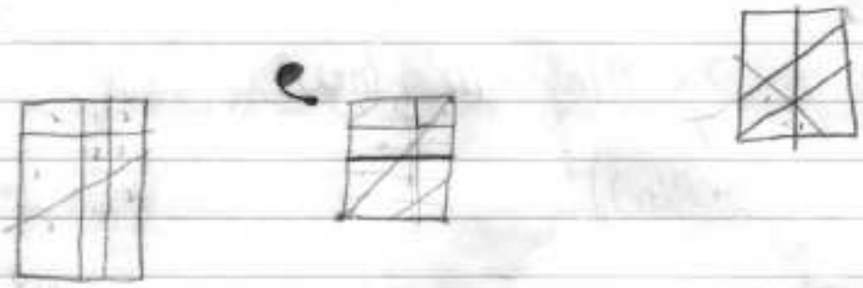
Problem 1



Pf Induction on the number of lines

Bo $P(n)$ = You can color the paper w/ n lines w/ 2 colors

Base case: $n=1$

This creates two regions. Make each region a different color.

$\therefore$ $P(1)$ holds

Inductive case

assume $P(1)$ $P(n)$ holds to show $P(n+1)$

The last $n+1$ line drawn divides the paper into two regions.

We know each region can be colored by the inductive hypothesis

Each region that borders the $n+1$st line "matches" a region on the other side of the line because lines go from edge to edge.

IE each region on one side touches (and immerses constraints on) exactly one region on the other side.

Case I No two lines have a point of intersection on the new line

Then each region touching the line touches two other regions that touch the line, except the edges. This means the regions alternate in color along the line. By changing all colors on one side if necessary, the map can be colored.

Case II Two lines intersect and the point of intersection is the new line

Then there are two "matching" regions that touch the line at one point. These two regions impose no constraints on each other. By making them opposite colors, regions on either side will be opposite and it will be okay.