# Practice Quiz 1

**Problem 1 (6 points)** Consider the following proposition.

$$\forall x \forall y\ (x < y \rightarrow \exists z\ (x < z \wedge z < y))$$

**(a) (2 points)** Prove or disprove this proposition, assuming that the universe is the set of integers.

**(b) (4 points)** Write the negation of this proposition. Your solution may *not* use the $\neg$ symbol, but may use any of the relations $=$, $<$, $>$, $\leq$, or $\geq$.

**Problem 2 (10 points)** Define the matrix

$$M = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

Let $M^n$ denote the product of $n$ copies of the matrix $M$. Use ordinary induction to prove that for all $n \geq 1$, the upper right entry of $M^n$ is $F_n$, the $n$-th Fibonacci number. State your inductive hypothesis clearly.

For reference, the standard formula for the product of $2 \times 2$ matrices is given below.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix} 1g$$

Recall that $F_0 = 0$, $F_1 = 1$, $F_2 = 1$, $F_3 = 2$, $\ldots$ .

**Problem 3 (12 points)** The vertices of a directed graph can be partitioned into *strongly-connected components*. Two vertices $u$ and $v$ belong to the same strongly connected component if there is a path from $u$ to $v$ and a path from $v$ to $u$. A strongly connected component may consist of a single vertex.

Math Moose has noticed a strange phenomenon. He starts with a directed graph and partitions the vertices into strongly-connected components. He then removes all edges that