Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professors David Karger and Nancy Lynch
Handout 24
March 17, 2000

# Mini-Quiz 7

1. Write your name:

2. (Rosen, Section 1.8, Problem 13) Show that $2^n$ is $O(3^n)$ but that $3^n$ is not $O(2^n)$.

# Tutorial 7 Problems

**Problem 1** Riemann's Zeta Function $\zeta(k)$ is defined to be the infinite summation:

$$1 + \frac{1}{2^k} + \frac{1}{3^k} \cdots = \sum_{j \geq 1} \frac{1}{j^k}$$

Prove that

$$\sum_{k \geq 2} (\zeta(k) - 1) = 1$$

**Hint:** Recall from problem set 5, problem 2, that $\sum_{k=2}^{n} \frac{1}{k(k-1)} = 1 - \frac{1}{n}$.

**Problem 2 Linear Recurrences**

Math Moose has discovered that the fluctuation in the price of stocks in his portfolio (in particular the stock prices of start-up companies that have recently gone public) satisfy a particular temporal property. The price fluctuation at any given day is twice that of the previous day, plus four times that of the day before that, plus negative eight times the day before that. In order to test out his discovery Math Moose has performed a series of tests of the following nature: he buys one stock of a particular start-up company and evaluates the price fluctuation on each successive day. He has found that the fluctuation of the stock price the day he buys the stock is $f(0) = 0$, the next day it's $f(1) = 1$, and the day after that it's $f(2) = 2$. Given this information can you come up with a closed-form solution for the price fluctuation of start-up stocks as discovered by Math Moose?

**(a)** Determine the linear recurrence, $f(n)$, corresponding to the fluctuation in the price of start-up company stocks in day $n$. What are the boundary conditions of $f(n)$ as discovered by Math Moose?

**(b)** Determine the characteristic equation of the linear recurrence $f(n)$ and find it's roots. **Hint:** The characteristic equation has a repeated root.

**(c)** Determine the recurrence solution.

**(d)** After testing his theory out, Math Moose has discovered that he has to adjust his way of evaluating the fluctuation of the stock price of start-up company founded by MIT graduates. He has noticed that MIT start-ups outperform other start-ups. Thus, he adjusts his scheme so that the price fluctuation for MIT start-up companies is increased by a constant dollar amount equal to 3 dollars each day.

What is the resulting inhomogeneous linear recurrence corresponding to Math Moose's scheme?

**(e)** Find the particular solution to the resulting inhomogeneous linear recurrence.

**(f)** What is the complete solution to the resulting inhomogeneous linear recurrence?

**(g)** Do you believe Math Moose's scheme for determining the price fluctuation of start-up stocks?

## Problem 3 Pigeonhole Principle

**(a)** Let $A$ be any set of $n + 1$ numbers from the set $\{1, 2, \ldots, 2n\}$. Prove that there are always two numbers in $A$ such that one divides the other.

**(b)** Let $A$ be any set of $n$ numbers from the set $\{1, 2, \ldots, 2n\}$. Prove that there are **not** always two numbers in $A$ such that one divides the other.