Problem 1. The "sum mod 3" of an n-bit string $v_1 \ldots v_n$ where $v_i \in \{0, 1\}$ is the sum of the bits of the string mod 3. Let a_n denote the number of n-bit strings for which the sum of the bits is 0 mod 3, b_n the number of n-bit strings for which the sum of the bits is 1 mod 3, and c_n denote the number of n-bit strings for which the sum of the bits is 2 mod 3.

(a) Write a system of recurrences for a_n, b_n, and c_n and explain how you arrived at this system. **Solution:**

Let us call a string a type-0 string if the sum of its bits is 0 mod 3, a type-1 if the sum of its bits is 1 mod 3, and a type-2 if the sum of its bits is 2 mod 3.
An n bit string $v = v' \cdot v_n$ (where v' is length $n-1$ and $v_n \in \{0, 1\}$,) will be type i if and only if (1) either $v_n = 0$ and v' is type i or (2) $v_n = 1$ and v' is type j such that $j + 1 = i$ mod 3.
Thus, the system if recurrences will be:

$$(1)\ a_n = a_{n-1} + c_{n-1}$$

$$(2)\ b_n = a_{n-1} + b_{n-1}$$

$$(3)\ c_n = 2^n - (a_n + b_n)$$

(b) Specify boundry conditions. **Solution:**

$a_0 = b_0 = c_0 = 0$, $a_1 = b_1 = 1$, $c_1 = 0$.

(c) Generate a linear recurrence for b_n. **Solution:**

First, substitute the definition of c_n from equation (3) into equation (1), to get $a_n = a_{n-1} + 2^{n-1} - a_{n-1} - b_{n-1} = 2^{n-1} - b_{n-1}$. Now, substitute a_n in equation (2) to get the linear recurrence $b_n = 2^{n-2} - b_{n-2} + b_{n-1}$.

(d) Is your recurrence homogeneous or not? **Solution:**

not

(e) What is the associated characteristic equation of the linear recurrence generated? **Solution:**

$x^2 - x + 1 = 0$ SHAFI - unfortunately no solution over the reals... this boggles my mind