Supplement to Final Fall 99

Replace problem 3 in the final by the following problem on invariants.

Problem 3'[10 points] **Invariants**

(a) Give the definition of a *state invariant* of a state machine?

(b) Besides, the identically `false` and identically `true` state invariants, how many other logically distinct state invariants does the following machine have? Describe them. (Two invariants are logically distinct if there exists a state in which one holds and the other doesn't.)

State Machine Description:
States $Q ::= \{A, B, C, D, E, F, G, H\}$, transitions:

$$\{(A, B), (B, C), (C, A), (D, E), (E, F)(F, D), (C, D), (G, A), (B, H)\}.$$

Solution:

$\{ABCDEFH\}, \{DEF\}, \{H\}$. **endsoln**

Replace problem 9 in the final by the following problem on linear recurrence - same problem with an example .

problem 9'[15 points] **Linear Recurrence**

You just moved into your new house and you are throwing a housewarming party. You invite 5 people on the first day and tell them to invite their friends. Each newly invited person invites 2 people the day after they are invited.

(c) [5 points] Give a recurrence describing the total number of people invited by the nth day. This includes everyone – i.e the people you invited, and the people invited by the people you invited etc.

(E.g. On the first day 5 people were invited, by the second day 15 people were invited including the 5 people you invited on the 1st day and the 10 people they invited on the 2nd day, by the third day 35 people were invited etc).

Solution: You know that the number of people for each day will equal the number of people who were invited from the day before in addition to 2 new guests for each of the new people on the day before. T(n) = T(n-1) + 2 [T(n-1) - T(n-2)] = 3 T(n-1) - 2 T(n-2) **endsoln**

(d) [5 points] Describe initial conditions for the recurrence.

Solution: You know that T(0) = 0 since no one was invited on day 0 and T(1) = 5, since you invited 5 people on the first day. **endsoln**

(e) [5 points] Give a closed form expression equal to the number of people invited by the nth day.

Solution: Solving this recurrence $a^2 = 3a - 2$ $a^2 - 3a + 2 = 0$ $(a-2)(a-1) = 0$ $a = 2$ or 1

$T(n) = c2^n + b1^n = c2^n + b$

Plugging in the initial conditions, gives you two equations : $T(0) = 0 = c + b$ $T(1) = 5 = c * 2 + b$ $c = 5, b = -5$

$T(n) = (5 * 2^n) - 5$

endsoln