Problems for Recitation 7

Problem 1. Let S be the set of numbers $7, 77, 777, \ldots$. Prove that there is an element of S that is divisible by 1997.

Problem 2. Show that in any sequence of n integers, there is either an increasing subsequence of length $\sqrt{n}$ or a decreasing subsequence of length $\sqrt{n}$.

1. Show that every DAG has a chain of length $> \sqrt{n}$ or an antichain of size $\geq \sqrt{n}$.
2. Define a DAG based on a given sequence of numbers so that chains correspond to increasing subsequences and antichains correspond to decreasing subsequences.
3. Put the above together to solve the problem.