Problems for Recitation 4

1. *Efficiency of Euclid's algorithm.*

   (a) Show that Euclid's GCD algorithm uses at most $2\log_2 x$ modulo operations. [Use derived variables]

   (b) Let F_n denote the n^{th} Fibonacci number. Show that:

      i. If $2 \leq x \leq F_n$ then the algorithm uses at most $n-2$ modulo operations.

      ii. For $n \geq 2$, $F_n \geq (\frac{3}{2})^{n-2}$.

      iii. Using the previous two results show that Euclid's algorithm actually uses at most $1.71\log_2 x$ modulo operations if $x \geq 2$.

2. *Choosy mating.*

   (a) We have the mating problem again, but now both boys and girls are more choosy about their partners. That is, among a set of n boys and n girls, each boy is unwilling to drop below his k^{th} choice and each girl is unwilliling to below her m^{th} choice. So it is possible that some of them are not married. How many pairs can we be sure of having in general ?

   (b) What if only the boys were choosy, i.e., the girls are willing to marry anyone (they still have a ranked list of preferences), but the boys will not go below their k^{th} choice ? How many pairs will the algorithm from class (TMA) assure us ?