Problems for Recitation 2

1. *Handshakes.* I recently went to a party with my spouse; there were 11 other married couples there. A lot of handshaking happened (but of course no one shook hands with their spouse). At the end, it turned out that for every k from 0 to 22, someone other than me had shaken hands with exactly k people. How many people did I shake hands with? Prove your answer.

2. *Geometric mean vs Arithmetic mean.* Show that for any non-negative real numbers $x_1, x_2, \ldots, x_n$,

$$(x_1 x_2 \ldots x_n)^{\frac{1}{n}} \leq \left(\frac{x_1 + x_2 + \ldots + x_n}{n} \right)$$