Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professors Nancy Lynch and Srinivas Devadas

Handout 29
October 15, 1998

Problem Set 6

Due: Start of class on October 20, 1998.

Read sections 5.1 and 5.2 in Rosen.

Problem 1. Find closed-form solutions to the following homogenous linear recurrences.

(a) $x_n = 5x_{n-1} - 6x_{n-2} \quad (x_0 = 0, x_1 = 1)$

(b) $x_n = 5x_{n-1} - 8x_{n-2} + 4x_{n-3} \quad (x_0 = 1, x_1 = 2, x_2 = 3)$

Problem 2. Find closed-form solutions to the following inhomogenous linear recurrences.

(a) (Fibonacci Plus) $x_n = x_{n-1} + x_{n-2} + 1 \quad (x_0 = 0, x_1 = 1)$

(b) $x_n = 3x_{n-1} - 2x_{n-2} + n \quad (x_0 = 0, x_1 = 1)$

Problem 3. The 6.042 staff recently received the following email.

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From: grzzwk@xoros19.gammaquad.plnt
To: 6042-staff@theory.lcs.mit.edu
Subject: Pset 5 issue

Thank you for contributing the MergeSortX19 algorithm to our
``intellectually challenged'' society.  My, my-- we feel mentally
bloated by this sudden infusion of technological wizardry!

As a token of thanks, we launched a Doritofier missile on a straight
line toward Earth.  Suppose that p(t) denotes the distance between
Xoros 19 and the Doritofier after t days of flight.  At the moment of
launch, the Doritofier was here on Xoros 19, and so p(0) = 0.  After a
single day, the Doritofier had already traveled 100 units toward
Earth, so p(1) = 100.  In general, the position of the missle is given
by the following recurrence:

       p(t+1) = 1.99 * p(t) - p(t-1)

Earth happens to be 1010 units from Xoros 19.  Gotta run!  Seeya!

/Grzzwk
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(a) Find a closed-form solution to the recurrence for the distance between Xoros 19 and the Doritofier after t days of flight.

(b) The characteristic equation approach gives a solution to this recurrence involving complex numbers. This makes evalutation awkward. Give a closed-form solution that does not involve complex numbers. DeMoivre's Identity may be helpful:

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

(c) Does the Doritofier missle hit earth?