Problems for Tutorial 9

1 More Games with Monty

Let's play another game. Monty shows you two cards, each lying face down on a table. He tells you there's a number from the range $\{1, \dots, n\}$ written on the face of each. You pick a card, and Monty shows you the number written on it. He now tells you that you can either choose to stay with that number, or switch to the number written on the other card. You *win* if the number you end up with is the biggest of the two (that is, if either you originally picked the biggest and decided to stay, or if you originally picked the smallest and decided to switch). A randomized strategy for this game is a function that, for each number i, assigns a probability p_i of switching *given* that the first card you picked was i. Can you find a randomized strategy for playing this game that gives you more than an even chance of winning, given that Monty *knows* what strategy you are using (and thus can pick the numbers he presents to you in a malicious way)?

2 Monty Goes Infinite

- Now suppose the numbers from the problem above are drawn from the set of natural numbers, $\mathbb{N}$. Can you find a strategy for playing this game that gives you more than an even chance of winning?
- Now suppose the numbers from the problem above are drawn from the set of integers, $\mathbb{Z}$. Can you find a strategy for playing this game that gives you more than an even chance of winning?