Problems for Tutorial 8

1 Some Useful Identities

1. Let $A_1, \ldots, A_n$ be a collection of disjoint sets such that their union is equal to the sample space. Prove by induction that for any event B

   $P(B) = P(B \cap A_1) + \ldots + P(B \cap A_n)$

2. Use part 1 to show that if $A_1, \ldots, A_n$ is a disjoint collection that covers the sample space, then for any events B, C such that $P(C) \neq 0$

   $P(B \mid C) = P(B \cap A_1 \mid C) + ... + P(B \cap A_n \mid C)$

   Observe the useful corollary:

   $P(B \mid C) = P(B \cap A \mid C) + P(B \cap \bar{A} \mid C)$

2 Trial By Jury

In the OJ Simpson trial it was claimed by the defense that the fact that Simpson battered his wife should be excluded from the evidence as the fraction of wife batterers that murder their wives is 1/2000 (Dershowitz Lemma). In this problem we will condition this probability on the event that Nicol is dead and see that we obtain a somewhat different result.

We have the following statistical information:

- number of women = 125M
- number of women murdered in 1994 = 6000
- number of women murdered by souse in 1994 = 1500
- number of women with spouse in 1994 = 50M
- number of women battered by spouse in 1994 = 3M

Being murdered counts as being battered. Also, the event of being murdered by somebody else is not influenced by whether the woman was being battered or not.

Define the events

G = Woman is murdered by spouse
M = Woman is murdered
B = Woman is battered

Use the given information to evaluate the following

- Dershowitz Lemma: $Pr(G \mid B) = 1/2000$
- $Pr(G \mid B \cap M)$
- $Pr(G \mid M)$

Do you think that the evidence that Simpson battered his wife should have been excluded based on Dershowitz Lemma?