Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Tutorial 3
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Problems for Tutorial 3

1 Running in the Family

Prove that in the last 1000 years you have had an ancestor A and an ancestor B such that A is decended from B on both sides, i.e. both of A's parents are descendants of B.

2 Uncomputable Functions

A computable function from the naturals to the naturals is a finite piece of code (in your favourite programming language). The code takes as input a natural number and outputs a natural number. Prove that not all functions f from the naturals to the naturals are computable.

3 A Number Theory Application

Suppose I have a collection of switches numbered 0 through n, all initially off. Someone comes and flips every switch (so they are all on). Another person then flips every second switch, starting with $0, 2, \ldots$. The next person flips every third switch, and so on until the last person flips only the flips numbered 0 and n. Which switches are on at the end? Prove it.