Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
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Tutorial 10
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Problems for Tutorial 10

1 Wald's Identity and Stopping Rules

Let $X_1, X_2 \dots$ be a sequence of iid random variables. A *stopping rule* for a set of random variables $\{X_n; n \geq 1\}$ is a random variable N with values in $\mathbb{N}$, such that the event $\{N \geq n\}$ may depend on $X_1, \dots, X_{n-1}$ but is independent of the values $X_n, X_{n+1}, \dots$.
Prove Wald's Identity for stopping rules:
If $X_1, X_2, \dots$ and N are as above, then

$$E[\sum_{i=1}^{N} X_i] = E[X]E[N]$$

(Hint: Let I_n be the indicator of the event $\{N \geq n\}$ and consider the sum $\sum_{n=1}^{\infty} X_n I_n$)

2 The Distressed Student

A Harvard student, lost in the underground corridors of MIT, finds himself in a room containing three doors. Door 1 leads him to freedom (out of MIT) after two hours travel; door 2 returns him to the his room after four hours travel, and door 3 returns him to the room after eight hours travel. Each time he finds himself in the room, the Harvard student picks a door with equal probability. Let T denote the time until he finds the exit.

(a) Define a sequence of iid random variables X_i and a stopping rule N such that

$$T = \sum_{i=1}^{N} X_i$$

(b) Use Wald's Identity to find $E[T]$

(c) Now suppose that an MIT student is also lost and finds herself in a similar situation to the Harvard student. The MIT student never choses the same door twice. What is $E[T]$ for the MIT student?