1 Wald's Identity

Let I_n be the indicator of $\{N \geq n\}$. Let

$$S_N = \sum_{n=1}^{N} X_x$$

Note that since $I_n = 1$ if $N \geq n$ and $I_n = 0$ if $N \leq n-1$ the above sum can be written as

$$S_N = \sum_{n=1}^{\infty} X_n I_n$$

Then we have

$$E[S_N] = \sum_{n=1}^{\infty} E[X_n I_n] = E[X] \sum_{n=1}^{\infty} E[I_n] = E[X] \sum_{n=1}^{\infty} P(N \geq n) = E[X]E[N]$$

2 Problem 2

(a) Let X_i take the values $2, 4, 8$ with probability $1/3$, and let $N = \min\{n | X_n = 2\}$. N is geometric with mean 3

(b) $E[T] = E[N]E[X] = 3(14/3) = 14$

(c) $E[T] = (1/3)(2) + (1/3)(4 + (1/2)(2 + 10)) + (1/3)(8 + (1/2)(2 + 6)) = 8$