Graph models

Problem 1. Suppose G is a graph with v vertices and e edges whose smallest degree is d_1 and whose largest degree is d_2. Show that $d_1 \leq 2e/v \leq d_2$.

Problem 2. Show that in any simple graph there are always two vertices with exactly the same degree.

Problem 3. **(a)** If G is a graph with 25 edges and each degree is at least four, what is the maximum number of vertices that G can have?

(b) If G is a graph with 50 edges, what is the least number of vertices that G can have?

Path and connectedness

Problem 4. **(a)** Let G be a bipartite graph. Show that the vertices of G can be listed such that the adjacency matrix of G has the following form, where 0 represents a matrix whose entries are all 0, and A_{21} is the transpose of A_{12}.

(b) Show that if G is a bipartite simple graph with v vertices and e edges, then $e \leq v^2/4$.

(c) Describe an algorithm to decide whether a graph is bipartite.

Problem 5. Show that a connected graph with n vertices has at least $n-1$ edges.

Application of graph coloring

Problem 6. A computer science department has eight committees and professors $\{A, B, C, D, E, F, G, H, I, J\}$ are assigned to these committees as follows:

Computing	$\{A, B, C, D\}$	*Advising*	$\{D, E\}$
Library	$\{A, E, F, G\}$	*Curriculum*	$\{C, A\}$
Recruitment	$\{G, H, I, J\}$	*Fund Raising*	$\{I, G\}$
RTP	$\{H, B, F\}$	*Education*	$\{J, C\}$

If each committee is scheduled to meet for one hour, what is the smallest number of hours required to schedule all eight committee meetings so that every professor is able to meet with all the committees to which he or she is assigned?