Problem Set 1

Due: Start of Class, Tuesday, September 10.

Reading: Sections 1.1 to 1.3 and 3.1 of *Discrete Mathematics and its Applications.*

Problem 1. Let p, q, and r be the following propositions:

p : You get an A on the final exam.
q : You do every exercise in this book.
r : You get an A in this class.

Write the following propositions using p, q, and r and logical connectives (e.g. $\vee$, $\wedge$, $\neg$, $\Rightarrow$).

(a) To get an A in this class, it is sufficient for you to get an A on the final.

(b) You do every exercise in this book, but you don't get an A on the final; nevertheless, you get an A in this class.

(c) Getting an A on the final and doing every exercise in this book is necessary for getting an A in this class.

(d) You will get an A in this class if and only if you either do every exercise in this book or you get an A on the final.

Problem 2. Find a compound proposition involving the propositions p, q, and r that is false when exactly two of p, q, and r are true. The compound propositon is true otherwise.

Problem 3. Let $Q(x, y)$ be the statement "x has been a contestant on y." Express each of the following sentences in terms of $Q(x, y)$, quantifiers and logical connectives, where the universe of the discourse for x is the set of all students at your school and for y is the set of all quiz shows on television.

a) There is a student at your school who has been a contestant on a television quiz show.

b) No student at your school has ever been a contestant on a television quiz show.

c) Every television quiz show has had a student from your school as a contestant.

d) There is a student at your school who has been a contestant on *Jeopardy* and on *Wheel of Fortune*.

e) At least two students from your school have been contestants on *Jeopardy*.

Problem 4. Let $F(x, y)$ be the statement "x can fool y," where the universe of discourse for both x and y is the set of all people in the world. Use quantifiers to express each of the following statements:

a) Everybody can fool somebody.

b) There is no one who can fool everybody.

c) Everybody can be fooled by somebody.

d) Nancy can fool exactly two people.

e) There is exactly one person whom everybody can fool.

f) No one can fool himself or herself.

Problem 5. Let $P(x), Q(x)$, and $R(x)$ be the statements "x is a clear explanation," "x is satisfactory," and "x is an excuse," respectively. Express each of the following statements using quantifiers; logical connectives; and $P(x), Q(x)$, and $R(x)$, where the universe of discourse is the set of all English text.

a) All clear explanations are satisfactory.

b) Some excuses are unsatisfactory.

c) Some excuses are not clear explanations.

d) Does c) follow from a) and b)? If not, is there a correct conclusion?

Problem 6. Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x (P(x) \vee Q(x))$ are not logically equivalent.

Problem 7. Determine whether each of the following arguments is valid. If an argument is correct, what rule of inference (from Rosen, section 3.1) is being used? If it is not, what fallacy (from Rosen) occurs? If an argument is not correct, provide the correct argument.

(a) *Theorem:* if $n^2 > 1$, then $n > 1$. *Proof:* If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$, then $n > 1$.

(b) *Theorem:* The number $\log_2 3$ is irrational. *Proof:* A number is irrational if it is not the ratio of two integers. Therefore, since $\log_2 3$ cannot be written in the form a/b where a and b are integers, it is irrational.

(c) *Theorem:* If $n^2 \leq 4$ then, $n \leq 2$. *Proof:* Because if n is a real number with $n > 2$, then $n^2 > 4$.

(d) *Theorem* If $n \leq 3$, then $n^2 \leq 9$. *Proof:* Because if n is a real number with $n > 3$, then $n^2 > 9$.

Problem 8. Prove that the product of an irrational number and a non-zero rational number is irrational using a proof by contradiction.

Problem 9. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Problem 10. Prove by cases that if x and y are real numbers, then $\max(x, y) + \min(x, y) = x + y$.