Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professors Nancy Lynch and Tom Leighton

Handout 33
November 19, 1996

Problem Set 11

Due: Start of Class, Tuesday, November 26, 1996.

Reading: Lecture Notes on *Probability.*

Problem 1. [15 points]

What is the probability of getting exactly 9 cards of the same suit in a bridge hand (which consists of 13 randomly chosen cards)? Define a "5-card major" to be a hand with either exactly 5 spades or exactly 5 hearts or both. What is the proabability of getting a 5-card major?

Problem 2. [20 points]

In the lottery game Keno, a participant selects 20 different numbers from the set $S = \{1, \ldots, 80\}$. To determine winners, the lottery randomly selects 10 different numbers from S, and anyone who selected those 10 numbers (among his or her 20 numbers) wins.

(a) What is the probability of winning Keno? (You may leave factorials and powers in your answer.)

(b) Suppose each of the 142 students in 6.042 selects number to play Keno. We don't know how students will select their numbers; in particular, they may not be selected uniformly or independently. Give upper and lower bounds on the probability that there is at least one winner in the class.

(c) Same as the last part, but say you know that exactly k distinct tickets (sets of 20 numbers) were chosen, and that the ith different ticket was chosen by exactly p_i people.

(d) Same as part **(b)**, but determine bounds on the probability of there being *exactly* one winner.

(e) Same as the last part, but again exactly k distinct tickets were chosen, and exactly p_i people chose the ith ticket.

Problem 3. [15 points]

A biased coin has probability p of heads and probability $1 - p$ of tails. Suppose we flip this coin n times.

(a) What is the probability of getting exactly k heads?

(b) For what values of k is the probability of getting exactly k heads greater than the probability of getting exactly $k-1$ heads? For what values of k is it less? What value of k is the most likely number of heads?

(c) Show that if $p = 1/n$ then the probability of getting no heads is $\sim 1/e$ (for large n). What is the ratio of the probability of no heads to the probability of exactly one head?

Problem 4. **[10 points]**

Show that if two events A and B are independent, then the events A and $\overline{B}$ are also independent.

Problem 5. **[10 points]**

Show that if two random variables R and T are independent, then R^2 and e^T are also independent.

Problem 6. **[15 points]**

Use the formulas derived in class to compute upper bounds on the following probabilities:

(a) In a poll with 2000 people, what is the probability that the result is off by more than 1%?

(b) In 1000 independent coin flips, what is the probability of getting ≥ 600 heads?

(c) In a noisy communication channel with 10% error rate, what is the probability that out of 1000 transmitted bits, ≥ 200 bits are wrongly transmitted (*i.e.*, `1` is received as `0` and vice versa)?

Problem 7. **[15 points]**

Let D_1 and D_2 denote random variables for two independent dice experiments. Let $M_1 = max(D_1, D_2)$, and $M_2 = min(D_1, D_2)$ be two random variables. Compute the distribution function for M_1. Are M_1 and M_2 independent random variables? Justify your answer.