Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professors Nancy Lynch and Tom Leighton

Handout 30
November 14, 1996

Problem Set 10

Due: Start of Class, Tuesday, November 19, 1996.

Reading: Lecture Notes on *Probability*.

Problem 1. [20 points]

Consider playing the *Lets Make a Deal*, where there are three prizes: a "Gift Certificate" from Toscanini, a "Nerd Pocket Protector", and a "Nerd Pride Button". Your goal is to win the gift certificate.

Suppose you (the contestant) happen to know that Carol always reveals the "Nerd Pocket Protector", if you did not choose the box covering it.
Does this additional information change the odds of winning for you? Is there a better strategy? By how much (if any) does this information increase your chances of winning?

Problem 2. [20 points]

Consider playing the *Lets Make a Deal* game with the following additional information.

(a) You are playing according to the original assumptions, but before you make your initial choice, you happen to see Carol placing a "Nerd Pride Button" underneath Box 1. What should you do to try to win the prize (the ice cream)? What is the probability that you win?

(b) Suppose Monty is determined to make you lose. In particular, if you initially choose a losing door, he may decide to end the game right there, opening your door to reveal the goat you have "won", never giving you the chance to switch doors. What is the probability of winning if you play the "don't-switch strategy"? What if you play the "switch strategy"?

Problem 3. [20 points]

In a complex system with 1,000 different components, each component is known to fail with probability at most $1/10,000$. Determine an upper bound on the probability that one or more components in the system fail.

Problem 4. **[20 points]**

A witness to a crime says that the criminal had a beard. We know from experience that if a criminal has a beard, witnesses will say so with probability 0.8, and if the criminal does not have a beard, witnesses will say that he does with probability 0.12. If 15% of the population has beards, what is the probability that a random criminal has a beard if the witness says he does?

Problem 5. **[20 points]**

(a) Prove that $Pr(\overline{A}|C) = 1 - Pr(A|C)$ for any events A and C.

(b) Is it possible for $Pr(A|B)$ and $Pr(A|C)$ to be very small, if $Pr(A|B \wedge C)$ is large? Give a counterexample or a proof to justify your answer.