Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professors Nancy Lynch and Tom Leighton

Handout 27
November 5, 1996

Problem Set 9

Due: Start of Class, Tuesday, November 12, 1996.

Reading: Section 4.6 of *Discrete Mathematics and its Application* by Kenneth H. Rosen, and lecture notes on *Generating Functions*.

Problem 1. **[10 points]**

Do Exercise 4.6 – 16 of Rosen.

Problem 2. **[8 points]**

Do Exercise 4.6 – 26 of Rosen.

Problem 3. **[8 points]**

Do Exercise 4.6 – 30 of Rosen.

Problem 4. **[10 points]**

Do Exercise 4.6 – 42 of Rosen.

Problem 5. **[10 points]**

Do Exercise 4.6 – 48 of Rosen.

Problem 6. **[24 points]**

Compute the (ordinary) generating function for each of the following sequences.

(a) $\langle 1, 1, 1, 1, 1, \ldots \rangle$

(b) $\langle 1, 1, 1, 0, 0, 0, 0, 0, 0, \ldots \rangle$

(c) $\langle 0, 0, 1, 1, 1, 1, 1, \ldots \rangle$

(d) $\langle 0, 0, 0, 1, 1, 1, 1, 1, \ldots \rangle$

(e) $\langle 0, 0, 0, 1, 1, 2, 1, 1, 1, 1, 1 \ldots \rangle$

(f) $\langle 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, \ldots \rangle$

(g) $\langle 1, 4, 9, 16, 25, \ldots \rangle$

(h) $\langle 2, 6, 12, 20, 30, 42, 56, \ldots \rangle$

(i) $\langle a_k \rangle = \left\langle k^2(k+2) \right\rangle$

(j) $\langle a_k \rangle = \left\langle 3^k + 5^k + 7^k \right\rangle$

(k) $\langle a_k \rangle = \langle 1/k! \rangle$

(l) $\langle a_k \rangle = \left\langle \sum_{i=0}^{k} \frac{1}{i!} \right\rangle$

Problem 7. **[12 points]**

Find partial fraction decompositions for the following functions.

(a) $\dfrac{8x}{4x^2 - 1}$

(b) $\dfrac{x^4 - x^3 - 6x^2 + 6x + 7}{x^2 + 3x + 2}$

(c) $\dfrac{x - 5}{x^3 + x^2 - x - 1}$

Problem 8. **[8 points]**

Use generating functions to solve the recurrence

$$a_n = \begin{cases} 2^n & \text{if } n < 3 \\ 2a_{n-2} + a_{n-3} & \text{if } n \geq 3 \end{cases}$$

Use induction to check your answer.

Problem 9. **[10 points]**

Let S be the multiset $\{\infty \cdot a_1, \infty \cdot a_2, \infty \cdot a_3, \infty \cdot a_4\}$. For each of the following problem parts, determine the generating function for the sequence $\langle g_0, g_1, \ldots \rangle$, where g_r is the number of r-combinations of S with the specified added restriction.

(a) Each a_k occurs an odd number of times.

(b) Each a_k occurs a multiple of 3 times.

(c) The element a_1 does not occur, and a_2 occurs at most once.

(d) The element a_1 occurs 1, 3, or 11 times, and the element a_2 occurs 2, 4, or 5 times.

(e) Each a_k occurs at least 10 times.