Problem Set 6

Due: Start of Class, Tuesday, October 22.

Reading: Sections 1.7–1.8 of *Discrete Mathematics and its Applications* by Kenneth H. Rosen, Chapters 2–3 of CLR (Handout 17).

Problem 1. [15 points]

A college loan officer offers you the following deal. He will give you $22,000 at the start of each year for four years starting today to pay for your college tuition. In return, you must pay $10,000 at the start of each year for twelve years starting in four years. The annual interest rate is 8%. Is this a fair deal?

Problem 2. [15 points]

Find a closed form expression $f(n)$ for $\displaystyle\sum_{i=1}^{n} i^2 x^i$.

Problem 3. [15 points]

Show that $\sum_{k=1}^{\infty} 1/k^{3/2}$ is bounded above by a constant.

Problem 4. [15 points]

Let $0 < x < 1$ and $0 < y < 1$. Evaluate the following infinite sum.

$$\sum_{i=1}^{\infty}\sum_{j=1}^{\infty} \frac{ij}{2} x^{i/2} y^{j+i/2}$$

Problem 5. [15 points]

(a) Determine whether $\log(n!) = O(n \log n)$. Justify your answer.

(b) Determine whether $\log(n!) = \Theta(n \log n)$. Justify your answer.

Problem 6. [10 points]

Suppose that $f(x) = O(g(x))$. Does it follow that $2^{f(x)} = O(2^{g(x)})$?

Problem 7. **[15 points]**

Show that

(a) $\lfloor x + 1/2 \rfloor = \Theta(x)$.

(b) $\log(x^2 + 1) = \Theta(\log_2 x)$.

(c) $\log_{10} x = \Theta(\log_2 x)$.