Quiz 2

- This quiz is open book.
- Put your name on the top of each page—the pages of this booklet will be separated for grading.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please do not put part of the answer to one problem on the back of the sheet for another problem.
- Do not spend too much time on any problem. Read them all through first and attack them in the order that allows you to make the most progress.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. **Be neat**.
- This quiz ends at 9 P.M. It contains six problems. The problems are 16 points each, and you will get 4 points for writing your name, tutorial TA and time clearly.
- GOOD LUCK!

Problem	Points	Grade
name	4	
1	16	
2	16	
3	16	
4	16	
5	16	
6	16	
Total	100	

Name: ____________________

Please circle your tutorial TA: Jennifer Parry Steve Ravi

Please circle your tutorial time: 10 11 12 1 2 3

Problem 1. [16 points]

Prove that if

$$\Lambda \cup \{\psi\} \models \chi \text{ and } \Lambda \cup \{\phi\} \models \chi \text{ and } \Lambda \models \psi \vee \phi$$

then

$$\Lambda \models \chi$$

Problem 2. [16 points]

Prove or disprove: $f(n) = O(g(n))$ implies $\lg(f(n) = O(\lg(g(n)))$, where $g(n)$ is nondecreasing, $g(1) > 1$ and $f(n) \geq 0$ for all sufficiently large n.

Problem 3. [16 points]

Solve the following recurrence:

$$f(1) = f(2) = 1$$

$$f(n) - 4(n-1) + 4(n-2) = 3^n$$

Problem 4. [16 points]

Give tight (Θ) bounds for $T(n)$. Assume that $T(n)$ is constant for $n \leq 2$.

(a) $T(n) = 7T(n/3) + n^2$

(b) $T(n) = T(2n/3) + T(n/4) + n$

(c) $T(n) = 4T(n/3) + n$

Problem 5. [16 points]

(a) How many ways are there to color n (distinguishable) balls using exactly k colors? (Use no more and no fewer than k colors)

(b) If $x > k$ colors are available, how many ways are there to color the n balls using exactly k of those colors?

(c) Prove that

$$x^n = \sum_{k=1}^{n} S(n,k) \cdot x(x-1)\cdots(x-k+1)$$

by showing that the two sides agree for any positive integer x. ($S(n,k)$ is the Stirling number of the second kind.)

Problem 6. [16 points]

How many permutations of ABCDEFG are there that don't contain "$\cdots$BEG$\cdots$" or "$\cdots$AD$\cdots$"?