Suppose I have a collection of switches numbered 0 through n, all initially off. Someone comes and flips every switch (so they are all on). Another person then flips every second switch, starting with $0, 2, \ldots$. The next person flips every third switch, and so on until the last person flips only the flips numbered 0 and n. Which switches are on at the end? Prove it.