A Cast the following into propositional symbols, and verify or refute:

> If Superman were able and willing to prevent evil,he would do so. If Superman were unable to prevent evil, he would be impotent. If he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, superman does not exist.

Bona replaced this problem with that of showing that in the series $7, 77, 777, \ldots$ there is a number divisible by 1993.

B Prove: If the numbers 1–10 are placed in a circle in any order, there will be 3 consecutive numbers that sum to at least 17. You might want to start by proving a lemma: at least one of any list of real numbers $a_1, \ldots, a_n$ is greater than or equal to its average. (Garrison Keeler, host of Prarie Home companion, opens his show with: "Coming to you from the shores of Lake Woebegone, where all the women are strong, all the men are good looking, and all the children are above average." Point out the problem.)

If you finish early, consider the following two (which some students saw already in recitation—just check with your group).

- There exist irrational numbers a and b such that a^b is rational (hint: consider $\sqrt{2}^{\sqrt{2}}$).
- $1 = -1$:

$$\begin{aligned}
\sqrt{-1} &= \sqrt{-1} \\
\sqrt{\frac{1}{-1}} &= \sqrt{\frac{-1}{1}} \\
\frac{\sqrt{1}}{\sqrt{-1}} &= \frac{\sqrt{-1}}{\sqrt{1}} \\
\sqrt{1} \cdot \sqrt{1} &= \sqrt{-1} \cdot \sqrt{-1} \\
1 &= -1
\end{aligned}$$