Practice Quiz 1

- This quiz is open book.
- Put your name on the top of each page—the pages of this booklet will be separated for grading.
- Write your solutions in the space provided. If you need more space, write on the back of the sheet containing the problem. Please do not put part of the answer to one problem on the back of the sheet for another problem.
- Do not spend too much time on any problem. Read them all through first and attack them in the order that allows you to make the most progress.
- Show your work, as partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. **Be neat.**
- This practice quiz contains nine problems. (THE REAL QUIZ WILL CONTAIN FEWER PROBLEMS) The problems are 11 points each, and you will get 1 point for writing your name clearly.
- GOOD LUCK!

Problem	Points	Grade
name	1	
1	11	
2	11	
3	11	
4	11	
5	11	
6	11	
7	11	
8	11	
9	11	
Total	100	

Problem 1-1. [11 points] Prove by induction that $3|(n^3 + 2n)$ for all $n \in \mathbb{N}$.

Problem 1-2. [11 points] Prove that all rectangles with integer dimensions, except for the 1×1 rectangle, can be tiled with tiles of dimension 1×2 and 1×3.

Problem 1-3. [11 points]

(a) Use Euclid's algorithm to find the *gcd* of 221 and 323.

(b) Find an integer solution to $221x + 323y = 187$.

Problem 1-4. [11 points] Show that in any set of 10 integers, there are 4 integers a_1, a_2, a_3 and a_4 such that $3|(a_i - a_j)$ for all $i, j \in \{1, 2, 3, 4\}$.

Problem 1-5. [11 points] Let $f : A \to B$ and $g : B \to C$ be functions. Answer the questions below and justify your answers.

(a) If $g \circ f : A \to C$ is a surjection, are f and g necessarily surjections?

(b) If $g \circ f : A \to C$ is an injection, are f and g necessarily injections?

(c) If $g \circ f : A \to C$ is a bijection, are f and g necessarily bijections?

Problem 1-6. [11 points] Some real numbers have multiple decimal representations (e.g., $0.2999\ldots = 0.3000\ldots$). Let S be the set of all such real numbers. Is S countable or uncountable? Justify your answer.

Problem 1-7. [11 points] Define the poset $(\{1, 2, 3, 4, 5\}^2, \prec)$ where $(a, b) \prec (c, d)$ iff $a \leq c$ and $b \leq d$.

(a) Give a chain which is as long as possible.

(b) Prove that the above chain is the largest.

(c) If the above were a poset of tasks with dependencies how many time steps would it take to execute all the tasks given an arbitrary number of processors? (Assume each task takes one time step on any processor.)

(d) Identify all the minimum elements.

(e) Specify the elements of a largest antichain.

Problem 1-8. [11 points] The *complement* of an undirected graph $G = (V, E)$ is the graph $G^c = (V, E^c)$. That is, G^c contains exactly those edges that are not in G. Prove that if G is not connected then G^c is.

Problem 1-9. [11 points] Discover and prove (by induction) a formula for $\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n$.