Prove: $\forall n \geq k(P(n))$

1. (Base) $P(k)$
2. (Inductive step) $\forall n \geq k[(\forall i, k \leq i \leq n(P(n))) \Rightarrow P(n +$
   1. Fix $n \geq k$
   2. Assume $\forall i, k \leq i \leq n(P(i))$
   3. $P(n+1)$
   4. QED Imp
3. QED Str

Prove: $\forall n \geq k(P(n))$

1. (Base) $P(k)$
2. (Base) $P(k+1)$
3. (Inductive step) $\forall n \geq k+1[(\forall i, k \leq i \leq n(P(n))) \Rightarrow P$
    1. Fix $n \geq k+1$
    2. Assume $\forall i, k \leq i \leq n(P(i))$
    3. $P(n+1)$
    4. QED
4. QED

Prove: $\forall e \in F(P(e))$

1. (Base) $P(0)$
2. (Base) $P(1)$
3. (Inductive step) $\forall e, e' \in F(P(e) \wedge P(e') \Rightarrow P((e \wedge e'))$
4. (Inductive step) $\forall e, e' \in F(P(e) \wedge P(e') \Rightarrow P((e \vee e'))$
5. (Inductive step) $\forall e \in F(P(e) \Rightarrow P((\neg e))$
6. QED Struct. ind. or

Prove: $\forall t \in T(P(t))$

1. (Base) $P(n)$, where n is the single-node tree.
2. (Inductive step) $\forall t \in T(P(t) \Rightarrow P(makeleft(t))$
3. (Inductive step) $\forall t \in T(P(t) \Rightarrow P(makeright(t))$
4. (Inductive step) $\forall t, t' \in T(P(t) \wedge P(t') \Rightarrow P(makeboth($
5. QED Struct. ind. o

Prove: $\forall s \in S(P(s))$

1. (Base) $P(\lambda)$
2. (Inductive step) $\forall s \in S(P(s) \Rightarrow P(s0))$
3. (Inductive step) $\forall s \in S(P(s) \Rightarrow P(s1))$
4. QED Struct. ind. o

Prove: $\forall e \in F(P(e))$

1. (Base) $P(0)$ 0 has no paren

2. (Base) $P(1)$ No parens

3. (Inductive step) $\forall e, e' \in F(P(e) \land P(e') \Rightarrow P((e \land e'))$

   1. Fix $e, e' \in F$.

   2. Assume $P(e) \land P(e')$, that is, each of e and e' h same no. of left and right parens.

   3. $P((e \land e'))$, that is, the exp. $e \land e'$ has same no. of left and right parens. By ind. hyp. (one left and

   4. QED Implication, U

4. (Inductive step) $\forall e, e' \in F(P(e) \land P(e') \Rightarrow P((e \lor e'))$ Similar to the

5. (Inductive step) $\forall e \in F(P(e) \Rightarrow P((\neg e))$ Similar to the

6. QED Struct. ind. or

Prove: $\forall t \in T(P(t))$

1. (Base) $P(n)$, where n is single-node tree. 0 edges, 1 no
2. (Inductive step) $\forall t \in T(P(t) \Rightarrow P(makeleft(t))$
    1. Fix t.
    2. Assume $P(t)$, that is, $|\mathit{edges}(t)| + 1 = |\mathit{nodes}(t)|$.
    3. $P(\mathit{makeleft}(t))$, that is, $|\mathit{edges}(\mathit{makeleft}(t))| + 1 = |\mathit{nodes}($
        1. $|\mathit{edges}(\mathit{makeleft}(t))| = |\mathit{edges}(t)| + 1$
        2. $|\mathit{nodes}(\mathit{makeleft}(t))| = |\mathit{nodes}(t)| + 1$.
        3. QED Algebra, 2.2,
    4. QED Implication,
3. (Inductive step) $\forall t \in T(P(t) \Rightarrow P(makeright(t))$ Similar to pr
4. (Inductive step) $\forall t, t' \in T(P(t) \wedge P(t') \Rightarrow P(makeboth(t, t'))$
    1. Fix t.
    2. Assume $P(t) \wedge P(t')$, that is, $|\mathit{edges}(t)| + 1 = |\mathit{nodes}(t)|$
       and $|\mathit{edges}(t')| + 1 = |\mathit{nodes}(t')|$.
    3. $P(\mathit{makeboth}(t, t'))$, that is, $|\mathit{edges}(\mathit{makeboth}(t, t'))| + 1 = |$
        1. $|\mathit{edges}(\mathit{makeboth}(t, t'))| = |\mathit{edges}(t)| + |\mathit{edges}(t')| + 2$
        2. $|\mathit{nodes}(\mathit{makeboth}(t, t'))| = |\mathit{nodes}(t)| + |\mathit{nodes}(t')| + 1$.
        3. QED Algebra, 4.2,
    4. QED Implication,
5. QED Struct. ind. o

Prove: $\forall s \in S(P(s))$

1. (Base) $P(\lambda)$ No patterns of
2. (Inductive step) $\forall s \in S(P(s) \Rightarrow P(s0))$
    1. Fix s.
    2. Assume $P(s)$.
    3. $P(s0)$
        1. $num(01, s0) \leq num(10, s0) + 1$. ???
        2. If $s0$ ends in 0 then $num(01, s0) \leq num(10, s0)$. ???
        3. QED Conjunction
    4. QED Implication, U
3. (Inductive step) $\forall s \in S(P(s) \Rightarrow P(s1))$
    1. Fix s.
    2. Assume $P(s)$.
    3. $P(s1)$
        1. $num(01, s1) \leq num(10, s1) + 1$. ???
        2. If $s1$ ends in 0 then $num(01, s1) \leq num(10, s1)$. ???
        3. QED Conjunction
    4. QED Implication, U
4. QED Struct. ind. or

3. $P(s1)$

- 1. *num*$(01, s1) \leq$ *num*$(10, s1) + 1$.
  - 1. If s ends in 0 then *num*$(01, s1) \leq$ *num*$(10, s1) + 1$
    - 1. Assume s ends in 0.
    - 2. *num*$(01, s) \leq$ *num*$(10, s)$ — ind. hyp. (3
    - 3. *num*$(01, s1) =$ *num*$(01, s) + 1$ — Adding one
    - 4. *num*$(10, s1) =$ *num*$(10, s)$
    - 5. *num*$(01, s1) \leq$ *num*$(10, s1) + 1$. — Algebra (3.
    - QED — Implication
  - 2. If s ends in 1 then *num*$(01, s1) \leq$ *num*$(10, s1) + 1$
    - ind. hyp. p
  - 3. If $s = \lambda$ then *num*$(01, s1) \leq$ *num*$(10, s1) + 1$.
    - $s1$ is just 1
  - 4. QED — Cases
- 2. If $s1$ ends in 0 then *num*$(01, s1) \leq$ *num*$(10, s1)$.
  - Vacuously
  - because
- 3. QED — Conjunction

3. $P(s0)$
    1. $\mathit{num}(01, s0) \leq \mathit{num}(10, s0) + 1.$ ind. hyp
    2. If $s0$ ends in 0 then $\mathit{num}(01, s0) \leq \mathit{num}(10, s0)$.
        1. $\mathit{num}(01, s0) \leq \mathit{num}(10, s0)$.
            1. If s ends in 0 then $\mathit{num}(01, s0) \leq \mathit{num}(10, s0$
               ind. hyp
            2. If s ends in 1 then $\mathit{num}(01, s0) \leq \mathit{num}(10, s0$
                1. Assume s ends in 1.
                2. $\mathit{num}(01, s) \leq \mathit{num}(10, s) + 1.$ ind. hyp
                3. $\mathit{num}(01, s0) = \mathit{num}(01, s)$
                4. $\mathit{num}(10, s0) = \mathit{num}(10, s) + 1$ Exactly
                5. $\mathit{num}(01, s0) \leq \mathit{num}(10, s0)$ Algebra
                6. QED Implicat
            3. If $s = \lambda$ then $\mathit{num}(01, s0) \leq \mathit{num}(10, s0)$.
               $s0$ is jus
               which
            4. QED Cases
        2. QED Propositional reasoning (truth table)
    3. QED Conjunction

1. (Base) $P(t_0)$, where t_0 is the single-node tree. By def., *num*
   and *numed*
2. (Inductive step) $\forall t \in T(P(t) \Rightarrow P(\textit{makeleft}(t))$
   1. Fix t.
   2. Assume $P(t)$, that is, $\textit{numedges}(t) + 1 = \textit{numnodes}(t)$.
   3. $P(\textit{makeleft}(t))$, that is, $\textit{numedges}(\textit{makeleft}(t)) + 1$
      $= \textit{numnodes}(\textit{makeleft}(t))$.
      1. $\textit{numedges}(\textit{makeleft}(t)) = \textit{numedges}(t) + 1$
         Definition of
      2. $\textit{numnodes}(\textit{makeleft}(t)) = \textit{numnodes}(t) + 1$
         Definition of
      3. QED Algebra
   4. QED
3. (Inductive step) $\forall t \in T(P(t) \Rightarrow P(\textit{makeright}(t))$ Similar to th
4. (Inductive step) $\forall t, t' \in T(P(t) \wedge P(t') \Rightarrow P(\textit{makeboth}(t, t'))$
   1. Fix t.
   2. Assume $P(t) \wedge P(t')$, that is, $\textit{numedges}(t) + 1 = \textit{numnodes}$
      and $\textit{numedges}(t') + 1 = \textit{numnodes}(t')$.
   3. $P(\textit{makeboth}(t, t'))$, that is, $\textit{numedges}(\textit{makeboth}(t, t')) + 1$
      $= \textit{numnodes}(\textit{makeboth}(t))$
      1. $\textit{numedges}(\textit{makeboth}(t, t')) = \textit{numedges}(t) + \textit{numedge}$
         Definition of
      2. $\textit{numnodes}(\textit{makeboth}(t, t')) = \textit{numnodes}(t) + \textit{numnod}$
         Definition of
      3. QED Algebra
   4. QED
5. QED Struct. ind.