Prove: $\forall n \geq 0(P(n))$

1. (Base) $P(0)$ There's exactly
   string of 0s a
2. (Inductive step) $\forall n \geq 0(P(n) \Rightarrow P(n+1))$
   1. Fix $n \geq 0$.
   2. Assume $P(n)$, that is, there are 2^n length n stri
   3. $P(n+1)$, that is, there are 2^{n+1} length $n+1$ s
      Every string of
      have either a
      attached at t
   4. QED Implication, UG
3. QED Induction

Prove: $\forall n \geq 0(P(n))$

1. (Base) $P(0)$
2. (Inductive step) $\forall n \geq 0[(\forall i, 0 \leq i \leq n(P(n))) \Rightarrow P(n -$
3. QED Str

Prove: $\forall n \geq k(P(n))$

1. (Base) $P(k)$
2. (Inductive step) $\forall n \geq k[(\forall i, k \leq i \leq n(P(n))) \Rightarrow P(n +$
3. QED Str

Prove: $\forall n \geq 12(P(n))$

1. (Base) $P(12)$ Use

2. (Inductive step) $\forall n \geq 12[(\forall i, 12 \leq i \leq n(P(n))) \Rightarrow P($

3. QED Str

2. (Inductive step) $\forall n \geq 12[(\forall i, 12 \leq i \leq n(P(n))) \Rightarrow P($

   1. Fix $n \geq 12$.
   2. $(\forall i, 12 \leq i \leq n(P(n))) \Rightarrow P(n+1)$
      1. Assume $\forall i, 12 \leq i \leq n(P(n)$
      2. $P(n+1))$
         1. $n = 12 \Rightarrow P(n+1)$ Use
         2. $n = 13 \Rightarrow P(n+1)$ Use
         3. $n \geq 14 \Rightarrow P(n+1)$???
         4. QED Cas
      3. QED Imp
   3. QED UG

3. $n \geq 14 \Rightarrow P(n+1)$

    1. Assume $n \geq 14$.
    2. $P(n-2)$ — Ind. hyp. 2.2.1 with $n - 2 \geq 12$
    3. $P(n+1)$ — Add one block to the the previous step.
    4. QED — Implication

Prove: $\forall n \geq 12(P(n))$

1. (Base) $P(12)$
2. (Base) $P(13)$
3. (Base) $P(14)$
4. (Inductive step) $\forall n \geq 14[(\forall i, 12 \leq i \leq n(P(n))) \Rightarrow P($
3. QED Very specialized stro

Prove: $\forall n \geq 0(P(n))$

1. (Base) $P(0)$ $F_0 = 0$, w
2. (Inductive step) $\forall n \geq 0[(\forall i, 0 \leq i \leq n(P(n))) \Rightarrow P(n -$

   ???
3. QED Strong in

Prove: $\forall n \geq 2(P(n))$

1. (Base) $P(2)$ F_2
2. (Base) $P(3)$ F_3
3. (Inductive step) $\forall n \geq 3[(\forall i, 2 \leq i \leq n(P(i))) \Rightarrow P(n+$
   1. Fix $n \geq 3$
   2. Assume $\forall i, 2 \leq i \leq n(P(i))$
   3. $P(n+1)$???
   4. QED Imp
4. QED Str

3. $P(n+1)$

   1. $F_n \geq (\frac{3}{2})^{n-2}$ Ind

   2. $F_{n-1} \geq (\frac{3}{2})^{n-3}$ Ind

   3. $F_{n+1} = F_n + F_{n-1}$ Def

   4. $F_{n+1} \geq (\frac{3}{2})^{n-2} + (\frac{3}{2})^{n-3}$ Plu
      i

   5. QED Alg

      1. $\frac{3}{2} + 1 \geq (\frac{3}{2})^2$ Ari

      2. $(\frac{3}{2})^{n-2} + (\frac{3}{2})^{n-3} \geq (\frac{3}{2})^{n-1}$ Alg

      3. $F_{n+1} \geq (\frac{3}{2})^{n-1}$ 3.3

      4. QED Def

Prove: $\forall n \geq 2(P(n))$

1. $P(2)$ Use
2. $\forall n \geq 2[(\forall i, 2 \leq i \leq n)(P(i)) \Rightarrow P(n)]$
   1. Fix $n \geq 2$
   2. Assume $\forall i, 2 \leq i \leq n(P(i))$, that is, all numbers
      and including n can be factored into primes
   3. $P(n+1)$, that is, $n+1$ can be factored into pr
      ???
   4. QED Imp
3. QED Strong induction

3. $P(n+1)$, that is, $n+1$ can be factored into pr
   1. If $n+1$ is prime, then $n+1$ can be factore
      Use n
   2. If $n+1$ is not prime, then $n+1$ can be fac
      1. Assume $n+1$ is not prime.
      2. $n+1$ can be written as $a \times b$, where 1
         Def. o
      3. a can be factored into primes. Strong
      4. b can be factored into primes. Strong
      5. $n+1$ can be factored into primes.
         Use fa
      6. QED Implica
   3. QED Cases

Prove: $\forall n \geq 3(P(n))$

1. (Base) $P(3)$ Trivial b
2. (Inductive step) $\forall n \geq 3(P(n) \Rightarrow P(n+1))$
    1. Fix $n \geq 3$
    2. Assume $P(n)$
    3. $P(n+1)$
        1. Fix T, a tournament, and $p_1 \to p_2 \to \ldots \to$
           an $n+1$-cycle in T.
        2. T contains a 3-cycle. ???
        3. QED Implicat
    4. QED Implicat
3. QED

2. T contains a 3-cycle.

   1. If $p_3 \rightarrow p_1$ then T contains a 3-cycle.

      Use $p_1 \rightarrow p_2 \rightarrow$

   2. If $p_1 \rightarrow p_3$ then T contains a 3-cycle.

      1. Assume $p_1 \rightarrow p_3$.
      2. T contains an n-cycle.

         Use $p_1 \rightarrow p_3 \rightarrow$

      3. T contains a 3 cycle.

         Inductive hypo

   3. QED — Cases

Prove: $\forall n \geq 1(P(n))$

1. (Base) $P(1)$ O
2. (Inductive step) $\forall n \geq 1[(\forall i, 1 \leq i \leq n(P(n))) \Rightarrow P(n$ -
   1. Fix $n \geq 1$
   2. Assume $\forall i, 1 \leq i \leq n(P(i))$.
   3. $P(n+1)$
      1. Fix any $n+1$-player tournament T.
      2. T has a locally fair ranking.
         1. Choose x to be any player,
            B the set of players that beat x, anc
            L the set of players that lose to x.
         2. B has a locally fair ranking. In
         3. L has a locally fair ranking. In
         4. Define a ranking R of all the players b
            B ranking, then x, then L ranking.
         5. R is a locally fair ranking for T.
         6. QED E
      3. QED U
   4. QED In
3. QED S

Prove: $\forall n \geq 1(P(n))$

1. (Base) $P(1)$ O
2. (Inductive step) $\forall n \geq 1[P(n) \Rightarrow P(n+1)]$
   1. Fix $n \geq 1$.
   2. Assume $P(n)$.
   3. $P(n+1)$.
      1. Fix T, an $n+1$-player tournament,
         and z, a player with the most wins.
      2. $\exists$ a locally fair ranking of T with z first. ?‍?
      3. QED U
   4. QED In
3. QED In

2. $\exists$ a locally fair ranking of T with z first.

 1. If z is undefeated in T then $\exists$ a locally fair ranking of T with z first.
 1. Assume z is undefeated in T.
 2. Choose R to be a locally fair ranking Prev
 3. z then R is a locally fair ranking of T.
 4. $\exists$ a locally fair ranking of T with z firs EG
 5. QED Impli
 2. If z is defeated by someone then $\exists$ a locally ranking of T with z first. ???
 3. QED Case

2. If z is defeated by someone then $\exists$ a locally fair ranking of T with z first.
   1. Assume z is defeated by someone.
   2. Choose x to be someone who defeated z.
   3. z has the best record in the subtournament involving all-bu
      z has best record in T, no wins are remo
   4. $\exists$ ranking of subtournament having z first.
      Inductive hypothesis, z has best record.
   5. Choose R to be a ranking of all-but-x having z first.
      EI
   6. In T, x is defeated by someone.
      z was defeated, and x's record in T is no better than z's.
   7. Define ranking S to be R, with x inserted right after the lowest-ranked player that beats x.
   8. S is a locally fair ranking of T.
      R is locally fair, x is beaten by the player ranked just above it and beats all players ranked below it.
   9. z is first in S. z is first in R; x is not inserted at the top
   10. $\exists$ a locally fair ranking of T with z first.
      S works, EG

   QED Implication