Prove: $\sqrt{2}$ is irrational.

1. $\sqrt{2}$ rational $\Rightarrow$ *false*
    1. Assume $\sqrt{2}$ rational.
    2. *false*
        1. Choose $a, b \in N^+$, $\sqrt{2} = \frac{a}{b}$, lowest terms. — Basic prope
        2. $2b^2 = a^2$ — Algebra
        3. $a^2$ is even. — EG, def. of
        4. $a$ is even. — ???
        5. Choose $c \in N$, $a = 2c$. — Def. of "ev
        6. $4c^2 = a^2 = 2b^2$ — Algebra
        7. $2c^2 = b^2$ — Algebra
        8. $b^2$ is even. — EG, def. of
        9. $b$ is even. — ???
        10. $a$ and $b$ not in lowest terms. — 1.2.4, 1.2.9
        11. QED — 1.2.1, 1.2.1
    3. QED — Implication
2. QED — Contradicti

The proof is by contradiction. Assume for purpose of diction that $\sqrt{2}$ is rational. Then we can write $\sqrt{2} = a/$ $a$ and $b$ are integers and the fraction is in lowest terms ing both sides gives $2 = a^2/b^2$ and so $2b^2 = a^2$. This that $a$ is even; that is, $a$ is a multiple of 2. As a result multiple of 4. Because of the equality $2b^2 = a^2$, $2b^2$ m be a multiple of 4. This implies that $b^2$ is even and so be even. But since $a$ and $b$ are both even, the fraction $a$ in lowest terms. This is a contradiction, and so the ass that $\sqrt{2}$ is rational must be false.

To prove: $\forall n \geq 0(P(n))$
It's enough to show:
1. (Base) $P(0)$
2. (Inductive step) $\forall n \geq 0(P(n) \Rightarrow P(n+1))$

To prove: $\forall n \geq k(P(n))$
It's enough to show:
1. (Base) $P(k)$
2. (Inductive step) $\forall n \geq k(P(n) \Rightarrow P(n+1))$

Prove: $\forall n \geq 0(1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2})$

Prove: $\forall n \geq 0(P(n))$

1. (Base) $P(0)$
2. (Inductive step) $\forall n \geq 0(P(n) \Rightarrow P(n+1))$
3. QED Ind

Prove: $\forall n \geq 0(\sum_{i=1}^{n} i = \frac{n(n+1)}{2})$.

1. (Base) $\sum_{i=1}^{0} i = \frac{0(0+1)}{2}$.
2. (Inductive step) $\forall n \geq 0(\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \Rightarrow \sum_{i=1}^{n+1} i =$
3. QED Ind

1. (Base) $\sum_{i=1}^{0} i = \frac{0(0+1)}{2}$ $(P(0))$
    1. $\sum_{i=1}^{0} i = 0$ — Def. of sum
    2. $\frac{0(0+1)}{2} = 0$ — Arithmetic
    3. QED — By 1.1 and 1

2. (Inductive step) $\forall n \geq 0 (\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \Rightarrow \sum_{i=1}^{n+1} i =$

   1. Fix $n \geq 0$.
   2. $\sum_{i=1}^{n} i = \frac{n(n+1)}{2} \Rightarrow \sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$
      1. Assume $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$
      2. $\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$ ???
      3. QED Imp
   3. QED UG

2. $\sum_{i=1}^{n+1} i = \frac{(n+1)(n+2)}{2}$

   1. $\sum_{i=1}^{n+1} i = (\sum_{i=1}^{n} i) + (n+1)$ Sum has one ex
   2. $(\sum_{i=1}^{n} i) + (n+1) = \frac{n(n+1)}{2} + n + 1$

      Inductive hypoth
   3. $\frac{n(n+1)}{2} + n + 1 = \frac{(n+1)(n+2)}{2}$ Algebra
   4. QED Combine previou

Prove: $\forall n \geq 0 \; (P(n))$

1. (Base) $P(0)$
2. (Inductive step) $\forall n \geq 0 \; (P(n) \Rightarrow P(n+1))$
    1. Fix $n \geq 0$
    2. $P(n) \Rightarrow P(n+1)$
        1. Assume $P(n)$
        2. $P(n+1)$ ???
        3. QED Implication
    3. QED Universal g
3. QED Induction

Prove: $\forall n \in N, n \geq 0$ $(6|n^3 - n)$

1. (Base) $6|0^3 - 0$
2. (Inductive step) $\forall n \geq 0(6|n^3 - n \Rightarrow 6|(n+1)^3 - (n+$
   1. Fix $n \geq 0$.
   2. Assume $6|n^3 - n$
   3. $6|(n+1)^3 - (n+1)$ ???
   4. QED Implication, UG (condensin
3. QED Induction

Prove: $\forall n \in N, n \geq 0(6|n^3 - n)$

1. (Base) $6|0^3 - 0$
2. (Inductive step) $\forall n \geq 0(6|n^3 - n \Rightarrow 6|(n+1)^3 - (n+$
   1. Fix $n \geq 0$.
   2. Assume $6|n^3 - n$
   3. $6|(n+1)^3 - (n+1)$
      1. $(n+1)^3 - (n+1) = n^3 - n + 3(n^2 + n)$ — Algebra
      2. $n^2 + n$ is even — Last lecture
      3. $2|n^2 + n$ — Definition of $|$
      4. $6|3(n^2 + n)$ — Multiplying both
      5. $6|(n^3 - n) + 3(n^2 + n)$ — Divides both term so divides sum.
      6. QED — 2.3.1, 2.3.5
   4. QED
3. QED

Prove: $\forall n \geq 0(\Sigma_{i=0}^{n} F_i^2 = F_n F_{n+1})$

1. (Base) $\Sigma_{i=0}^{0} F_i^2 = F_0 F_1$
2. (Inductive step) $\forall n \geq 0(\Sigma_{i=0}^{n} F_i^2 = F_n F_{n+1}$
$$\Rightarrow \Sigma_{i=0}^{n+1} F_i^2 = F_{n+1} F_{n+2}$$
3. QED

Prove: $\forall n \geq 0(\Sigma_{i=0}^{n} F_i^2 = F_n F_{n+1})$

1. (Base) $\Sigma_{i=0}^{0} F_i^2 = F_0 F_1$ Both s
2. (Inductive step) $\forall n \geq 0(\Sigma_{i=0}^{n} F_i^2 = F_n F_{n+1} \Rightarrow \Sigma_{i=0}^{n+1} F$
    1. Fix $n \geq 0$.
    2. Assume $\Sigma_{i=0}^{n} F_i^2 = F_n F_{n+1}$
    3. $\Sigma_{i=0}^{n+1} F_i^2 = F_{n+1} F_{n+2}$
        1. $\Sigma_{i=0}^{n+1} F_i^2 = \Sigma_{i=0}^{n} F_i^2 + F_{n+1}^2$ Sum h
        2. $\Sigma_{i=0}^{n} F_i^2 + F_{n+1}^2 = F_n F_{n+1} + F_{n+1}^2$ Induct
        3. $F_n F_{n+1} + F_{n+1}^2 = F_{n+1}(F_n + F_{n+1})$ Algebr
        4. $F_{n+1}(F_n + F_{n+1}) = F_{n+1} F_{n+2}$ Fibona
        5. QED Combi
    4. QED Implica
3. QED Induct

Prove: $\forall n \geq 3$ (Sum of interior angles of any $n$-sided

convex polygon is $(n-2)180$ degree

1. (Base) Sum of angles in any triangle is 180.
2. (Inductive step) $\forall n \geq 3$ $(P(n) \Rightarrow P(n+1))$
   1. Fix $n \geq 3$.
   2. Assume sum of angles of any $n$-sided convex po
   3. Sum of angles of any $n+1$-sided convex poly is
      1. Fix any $n+1$-vertex convex poly $X$,

         say with vertices $x_1, x_2, \ldots, x_{n+1}$
      2. Let $Y$ be poly with vertices $x_1, x_2, \ldots, x_n$ (c
      3. $Y$ is a convex poly with at least 3 vertices.
      4. Sum of angles of $Y$ is $(n-2)180$.
      5. Sum of angles of triangle $T = x_n, x_{n+1}, x_1$
      6. Sum of angles in $X$ = sum in $Y$ + sum in

         $= (n-2)180 + 180 = (n-1)180.$
      7. QED
   4. QED
3. QED

Prove: $\forall n \geq 0\ (P(n))$

1. (Base) $P(0)$
   1. $\forall k(f(0,k) = \frac{(0+k)!}{0!k!})$ Bot
   2. QED Def
2. (Inductive step) $\forall n \geq 0(P(n) \Rightarrow P(n+1))$
   1. Fix $n \geq 0$.
   2. Assume $P(n)$, that is, row $n$ is correct.
   3. $P(n+1)$, that is, row $n+1$ is correct. ???
   4. QED Imp
3. QED Ind

3. $P(n+1)$

  1. $\forall k(Q(n+1,k))$

    1. (Base) $Q(n+1,0)$

    2. (Inductive step) $\forall k \geq 0(Q(n+1,k) \Rightarrow$

    3. QED Inductio

  2. QED Definitio

1. (Base) $Q(n+1,0)$
   1. $f(n+1,0) = \frac{(n+1+0)!}{n+1!0!}$ Both sides ar
   2. QED Definition of

2. (Inductive step) $\forall k \geq 0(Q(n+1,k) \Rightarrow Q(n+1,k+1$

   1. Fix $k \geq 0$.
   2. Assume $Q(n+1,k)$.
   3. $Q(n+1,k+1)$
      1. $f(n+1,k+1) = f(n,k+1) + f(n+1,k)$ Definition of
      2. $f(n+1,k) = \frac{(n+1+k)!}{(n+1)!k!}$ Inductive hyp def. of $Q($
      3. $f(n,k+1) = \frac{(n+k+1)!}{n!(k+1)!}$ Inductive hyp def. of $P($
      4. $f(n+1,k+1) = \frac{(n+k+1)!}{n!(k+1)!} + \frac{(n+1+k)!}{(n+1)!k!} = \frac{(n}{(n}$ Combine equ
      5. QED Definition of
   4. QED Induction