Tutorial 5 Problems

Problem 1 Computing GCDs

Consider the following procedure for computing the greatest common divisor (gcd) of a non-empty list of positive integers $(x_1, x_2, \ldots, x_n)$.

If all entries in the list are equal, then the procedure outputs the common value and terminates. Otherwise, the procedure selects two entries with different values and replaces the larger one by the difference.

(a) Model this procedure as a state machine.

(b) Prove that the procedure terminates.

(c) Prove that the procedure computes the gcd of $x_1, x_2, \ldots, x_n$.

Problem 2 Strange motions

You are standing in the bottom left square of a $2n \times 12$ grid. It is 5 minutes before the hour and you are in a rush to get to your 6.042 tutorial room which is located in the bottom right square of the grid. Unfortunately, you are only allowed to move to another square if the distance between the centers of the squares is precisely $\sqrt{r}$. Can you make it to tutorial if:

(a) r is divisible by 2?

(b) r is divisible by 3 and $2n \not\equiv 1 \pmod 3$?

(c) $n = 20$ and $r = 73$?

(d) $r = 97$?

Problem 3 A Puzzle

Math Moose has been given a new puzzle: a 3×3 grid of squares, where each square contains one of the digits in the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, each digit appearing exactly once in the puzzle. Each row can be rotated right or left and each column can be rotated up or down. The effects of such rotations are demonstrated in the following figure.

The first picture shows the initial configuration of the puzzle. The leftmost column is then rotated downward to give the middle picture. The 2 and 7 move down and the 5 rotates back up to the

2	3	6		5	3	6		5	3	6
7	8	9	→	2	8	9	→	8	9	2
5	4	1		7	4	1		7	4	1

Figure 1: Examples of puzzle row and column rotations.

top. Then, the middle row is rotated leftward to produce the final picture. The 8 and 9 move left and the 2 rotates back around to the right.

Since discovering the puzzle, Math Moose has spent hours trying to obtain the following configuration, starting of course from the initial configuration of Figure 1 and using legal row and column rotations.

1	2	3
4	5	6
7	8	9

Sad to say, Math Moose has failed miserably. He is beginning to suspect that the puzzle might be designed in a way that makes this configuration impossible, no matter what sequence of row and column rotations one attempts.

Given the initial configuration of the puzzle indicated above, namely 236 789 541, prove that Math Moose can never obtain the configuration 123 456 789.