Problem Set 1

Due: 2:35pm in lecture, September 12, 2000.

Reading: Vellman, *How to Prove It*, Chapter 1; Sections 3.1, 3.2.
Rosen, *Discrete Mathematics and Its Applications*, Sections 1.1, 1.2, 1.4, 1.5, 2.3, 3.1
Some of this reviews elementary mathematical topics that should be familiar to you; you can skim these. Do look closely at any topics that seem unfamiliar, and if there are many such topics, we encourage you to discuss this with your TA so he can help you plan a catch-up strategy.

Problem 1 Propositional logic

(a) Rosen §1.1, Exercise 24 (f).

(b) Rosen §1.2, Exercise 8 (c),(d).

(c) Velleman §1.5, Exercise 6:

`Which of the following formulas are equivalent?`

`(a)` $P \to (Q \to R)$
`(b)` $Q \to (P \to R)$
`(c)` $(P \to Q) \land (P \to R)$
`(d)` $(P \land Q) \to R$
`(e)` $P \to (Q \land R)$

Problem 2 Set theory

(a) Rosen §1.4, Exercise 16 (a),(b),(c),(d).

(b) Rosen §1.5, Exercise 12(c).

(c) Rosen §1.5, Exercise 18 (a),(b),(c).

(d) Rosen §1.5, Exercise 36.

Problem 3 Proofs or fallacies?

Determine whether each of the following arguments is valid. If it is not valid, then explain what the error is. (You may want to use the rules of inference and fallacies from Rosen in your solutions.)

(a) *Theorem:* if $n^3 > 1$, then $n > 1$. *Proof:* If n is a real number such that $n > 1$, then $n^3 > 1$. Suppose that $n^3 > 1$, then $n > 1$.

(b) *Theorem:* The number $\log_2 3$ is irrational. *Proof:* A number is irrational if it is not the ratio of two integers. Therefore, since $\log_2 3$ cannot be written in the form a/b where a and b are integers, it is irrational.

(c) *Theorem:* If $n^2 > 9$ then, $n > 3$. *Proof:* Because if n is a real number with $n > 3$, then $n^2 > 9$.

Problem 4 Proofs Velleman §3.1, Exercise 6:
`Suppose x is a real number and` $x \neq 0$. `Prove that if` $\frac{\sqrt[3]{x}+5}{x^2+6} = \frac{1}{x}$ `then` $x \neq 8$.

Problem 5 And more proofs Rosen §3.1, Exercise 46.