Problem Set 8.5

You do not have to turn in the solutions to this Problem Set. It is designed to help you study for the second quiz.

Reading: Lecture notes 17-18.

Quiz 2: Wednesday, November 15 7-9pm. Covers lectures 11-18.

Problem 1 Vote today $6N$ voters show up to vote for one of three candidates: Bush, Gore and Nader.

(a) Assuming that voters are indistinguishable, what is the number of possible vote counts?

(b) What is the number of possible vote counts in which Nader wins the popular vote?

Problem 2 How many ways are there to distribute eight balls into six distinct boxes with the first two boxes having collectively at most four balls if

(a) the balls are identical?

(b) the balls are distinct?

Problem 3 How many ways can you choose n out of $2n$ objects, given that n of the $2n$ objects are identical?

Problem 4 How many different ways are there to list the numbers $1, 2, \ldots, 2n$ such that the even numbers appear in increasing order and the odd numbers appear in decreasing order?

Problem 5 We wish to prove the identity

$$\sum_{r=0}^{n} \binom{n}{r}^2 = \binom{2n}{n},$$

for $n \in \mathbb{N}$.

(a) Use the binomial theorem and the fact that

$$(1+x)^n(1+x)^n = (1+x)^{2n}$$

to prove the identity.

(b) Consider a set of $2n$ objects, of which n are red and n are blue. Give a combinatorial proof of the identity.

Problem 6 Prove or disprove the following statements about sets.

(a) The product of a finite number of countable sets is countable.

(b) A countable union of countable sets is countable.

Problem 7 Use a diagonalization argument to show that the set of functions from the positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is uncountable.