Massachusetts Institute of Technology | Handout 36
6.042J/18.062J: Mathematics for Computer Science | October 31, 2000
Professors Nancy Lynch and Srinivas Devadas

Problem Set 8

Due: 2:35PM in lecture, Tuesday, November 7, 2000.

Be sure to indicate your TA's name clearly on the solutions your turn in.

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Self-study

Read: Rosen Section 4.1 Self-study problems (not to be handed in): Exercises 1, 3, 5, 7, 9, 11, 15, 17

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Reading: Lecture notes; Rosen 4.1, 5,3, 5.5, 5.6

Policy on collaboration If you collaborated with someone on the problem sets, you should mention their names on top of the problem set. If you did not collaborate with anyone, you should explicitly say so. As a reminder, the write-up of your solutions should be entirely your own.

Problem 1 In a survey of 270 college students, it is found that:

- 64 like brussels sprouts
- 94 like broccoli
- 58 like cauliflower
- 26 like both brussels sprouts and broccoli
- 28 like both brussels sprouts and cauliflower
- 22 like both broccoli and cauliflower
- 14 like all three vegetables

How many students do not like ANY of the aforementioned vegetables?

Problem 2

(a) In how many ways can the natural numbers $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$ be arranged in a sequence so that no number is in its original position?

(b) In how many ways can the natural numbers $0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11$ be arranged in a sequence so that no *even* number is in its original position?

Problem 3 How many bit strings of length 10 contain either five consecutive 0s or five consecutive 1s?

Problem 4

(a) (5 points) Recall the double induction example from Lecture 3, which involved a function $f : \mathbb{N} \times \mathbb{N} \Rightarrow \mathbb{N}$ defined by: $f(n, 0) = 1$ for all n, $f(0, k) = 1$ for all k, and $f(n+1, k+1) = f(n+1, k) + f(n, k+1)$ for all n and k. We proved by double induction that $f(n, k) = \frac{(n+k)!}{n!k!}$ for all n and k.
Recall the definition of binomial coefficients: $\binom{a}{b} = \frac{a!}{(a-b)!b!}$. Express $f(n, k)$ in terms of binomial coefficients.

(b) (5 points) What identity about binomial coefficients arises from what we proved in Lecture 3? Hint: how can you express $\binom{n+1}{k+1}$ in terms of smaller binomial coefficients?

(c) Recall the definition of Fibonacci numbers:

$$F_0 = 0; F_1 = 1; \text{for every } n \geq 2, F_n = F_{n-2} + F_{n-1}$$

Prove that for all $n \geq 0$

$$F_{2n} = \sum_{i=0}^{n-1} \binom{2n-i-1}{i} \qquad \text{AND} \qquad F_{2n+1} = \sum_{i=0}^{n} \binom{2n-i}{i}$$

Hint: use induction, and the formula you derived above

Problem 5 On a set of n elements how many of the following are there?

(a) binary relations

(b) symmetric binary relations

(c) reflexive binary relations

(d) symmetric and reflexive binary relations

(e) symmetric or reflexive binary relations