Massachusetts Institute of Technology
6.042J/18.062J: Mathematics for Computer Science
Professors Nancy Lynch and Srinivas Devadas

Handout 33
October 27, 2000

Mini-Quiz 7

1. Write your name:

2. **Injections, Surjections, and Bijections**

Let g be a function from A to B, and f a function from B to C. The composition $f \circ g$ is defined by: $f \circ g(x) = f(g(x))$.

Determine whether each of the following statements is true or false. If it's true, prove it. If it's false, give a counterexample.

(a) If f and g are both injective then $f \circ g$ is injective.

Solution: Let x,y be any two distint elements in A. Since g is one-to-one (surjective), $g(x)$ and $g(y)$ are distinct elements in B. Since f is one-to-one, $f(g(x))$ and $f(g(y))$ are distince elements of C. Thus, $f \circ g$ is injective.

(b) If f and g are both surjective then $f \circ g$ is surjective.

Solution: Let y be an arbitrary element in C. Since f is surjective, $y = f(b)$ for some element $b \in B$. Since g is also surjective, there must exist an $x \in A$ such that $b = g(x)$. Therefore, for any element $y \in C$, there exists an element $x \in A$ such that $y = f(g(x))$. Thus, $f \circ g$ is surjective.