Tutorial 2 Problems

Problem 1

(a) Show that $(\forall x P(x)) \vee (\forall x Q(x))$ is not logically equivalent to $\forall x (P(x) \vee Q(x))$ (that is, that the universal quantifier does not distribute over OR).

(b) Show that the existential qualifier does distribute over OR.

Problem 2

(a) A false proof about 2n by 2n tilings.

Claim. *For $n \in \mathbb{N}, n \geq 1$, any $2n \times 2n$ square grid can be tiled by L-shaped tiles such that any particular square is left untiled.*

Proof. The proof is by strong induction on n. Let $P(n)$ be the predicate: "Any $2n \times 2n$ square grid can be tiled by L-shaped tiles such that any particular square is left untiled".

Base Case:

The base case, for $n = 1$, is very simple: just use one tile.

Inductive Step:

Assume that the claim is true for all square grids smaller than $2n$, and we prove it for the $2n \times 2n$ square grid. This follows the proof of the $2^n \times 2^n$ case: Given a $2n \times 2n$ square grid, with one square designated to be left untiled, we construct a tiling (using L-shaped tiles) for all the other squares, as follows: Divide the square grid up into four quadrants, each $n \times n$. By the inductive hypothesis (since $n < 2n$ for all $n \geq 1$), we can tile each of these quadrants (with the L-shaped tiles) completely, except for one square of our choice. For the quadrant with the square to be left untiled, choose that square. For the others, choose the squares at the corner where the quadrants meet. This forms an L-shaped space at the center of the $2n \times 2n$ square grid, which can be filled by a single L-shaped tile. Thus, the entire $2n \times 2n$ square grid is tiled, except for the square designated.

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Point out any logical errors in this proof (that is, not simply places where some steps were skipped, but where there is actually an error, though you are free to point out places where more steps should have been included for clarity). Then, either construct a correct proof, or show that the claim is false.

(b) Show that a 7 by 7 chessboard with one square removed can be completely covered using L-shaped tiles.

(c) Show that an n by n chessboard with one square removed can be completely covered using L-shaped pieces of the sort described in lecture, if $n > 5$, n is odd, and n is not divisible by 3.

Problem 3

(a) Prove that $2^0 + 2^1 + \cdots + 2^n = 2^{n+1} - 1$, for every $n \in N$.

(b) Come up with a formula for $3^0 + 3^1 + \cdots + 3^n$, and prove your formula.