Better Way to Count 2 Pair

counting 2-pair poker hands
to count, choose:
• set of 2 ranks for pairs
• suits for the smaller rank
• suits for the larger rank
• last card

counting 2-pair poker hands
example: choosing
{9, 4}, {♠, ♥}, {♦, ♠}, 3♣
specifies 2-pair hand:

4♦, 4♥
counting 2-pair poker hands

example: choosing 
\{9, 4\}, \{\♦, \♥\}, \{\♦, \♠\}, 3♠ 
specifies 2-pair hand: 
4♦, 4♥, 9♦, 9♠

Unique representation

Now given a 2-pair hand 
4♦, 4♥, 9♦, 9♠, 3♣ 

have unique sets of 
• two ranks: \{4, 9\} 
• suits for 4's: \{♦, ♥\} 
• suits for 9's: \{♦, ♠\} 
• unpaired card: 3♣

A Bijection

which shows that the map from 
[2-set of ranks] \times [2-set of suits] \times 
[2-set of suits] \times remaining card 
to \{2-pair hands\} 
is a bijection
counting 2-pair poker hands

to count, choose:
• set of 2 ranks for pairs $\binom{13}{2}$
• suits for the smaller rank $\binom{4}{2}$
• suits for the larger rank $\binom{4}{2}$
• last card $52 - 8 = 44$

Counting 2-pair poker hands

and so # 2-pair hands is

#rank 2-sets · #suit 2-sets ·
#suit 2-sets · #remaining cards

\[
\binom{13}{2} \cdot \binom{4}{2} \cdot \binom{4}{2} \cdot 44 \checkmark
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