Properties of relations

A relation from $A$ to $B$ is:

- a function if every element of $A$ is assigned to at most one element of $B$,
- total when every element of $A$ is assigned to at least one element of $B$,
- surjective if every element of $B$ is assigned to at least one element of $A$,
- injective if every element of $B$ is mapped at most once, and
- bijective if it is total, surjective, injective, and a function.

Properties of a relation on $A$:

Reflexivity $R$ is reflexive if

$$\forall x \in A. \ xRx.$$  
“Everyone likes themselves.”
Every node in $G$ has a loop.

Irreflexivity $R$ is irreflexive if

$$\neg \exists x \in A. \ xRx.$$  
“No one likes themselves.”
There are no loops in $G$.

Symmetry $R$ is symmetric if

$$\forall x, y \in A. \ xRy \Rightarrow yRx.$$  
“If $x$ likes $y$, then $y$ likes $x$.”
If there is an edge from $x$ to $y$ in $G$, then there is an edge from $y$ to $x$ in $G$ as well.

Antisymmetry $R$ is antisymmetric if

$$\forall x, y \in A. \ (xRy \land yRx) \Rightarrow x = y.$$  
“No pair of distinct people like each other.”
There is at most one directed edge between any pair of distinct nodes.

Transitivity $R$ is transitive if

$$\forall x, y, z \in A. \ (xRy \land yRz) \Rightarrow xRz.$$  
“If $x$ likes $y$ and $y$ likes $z$, then $x$ likes $z$ too.”
For any walk $v_0, v_1, \ldots, v_k$ in $G$ where $k \geq 2$, $v_0 \rightarrow v_k$ is in $G$ (and, hence, $v_i \rightarrow v_j$ is also in $G$ for all $i < j$).