Problems for Recitation 13

1 Asymptotic Notation

Which of these symbols \( \Theta \), \( O \), \( \Omega \), \( o \), \( \omega \) can go in these boxes? (List all that apply.)

\[
\begin{align*}
2n + \log n & = \Theta(n) \\
\log n & = \Theta(n) \\
\sqrt{n} & = \Theta(\log^{300} n) \\
n2^n & = \Theta(n) \\
n^7 & = \Theta(1.01^n)
\end{align*}
\]
2 Asymptotic Equivalence

Suppose \( f, g : \mathbb{Z}^+ \rightarrow \mathbb{Z}^+ \) and \( f \sim g \).

1. Prove that \( 2f \sim 2g \).
2. Prove that \( f^2 \sim g^2 \).
3. Give examples of \( f \) and \( g \) such that \( 2f \not\sim 2g \).
4. Show that \( \sim \) is an equivalence relation
5. Show that \( \Theta \) is an equivalence relation

3 More Asymptotic Notation

1. Show that
   \[(an)^{b/n} \sim 1.\]
   where \( a, b \) are positive constants and \( \sim \) denotes asymptotic equality. Hint \( an = a2^{\log_2 n} \).
2. You may assume that if \( f(n) \geq 1 \) and \( g(n) \geq 1 \) for all \( n \), then \( f \sim g \Rightarrow f^{1/n} \sim g^{1/n} \).
   Show that
   \[\sqrt[n]{n!} = \Theta(n).\]