Staff Solutions to Mini-Quiz 8, morning

Problem 1 (5 points).
Define the length of a planar embedding, $E$, of a graph $G$ to be the sum of the lengths of the faces of $E$.

(a) Give a formula for $E$ in terms of $v$, $c$, and $e$ (number of vertices, components, and edges in $G$). Hint: the length of face $abca$ is 3.

Solution. $E = 2e$.

(b) Conclude that all embeddings of the same planar graph have the same length.

Solution. Each edge has exactly two occurrences on the faces of an embedding, so the length of an embedding is always twice the number of edges in the graph, which is the same for any embedding of the same graph.

Problem 2 (5 points).
Show that this series converges:

$$
\sum_{i=1}^{\infty} \frac{1}{i} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots
$$

Hint: $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

Solution.

$$
\sum_{i=1}^{\infty} \frac{1}{i} = -1 + \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots = -1 + \frac{1}{2 \cdot 3} + \frac{1}{4 \cdot 5} + \cdots = -1 + \sum_{i=1}^{\infty} \frac{1}{(2i)(2i + 1)} < \sum_{i=1}^{\infty} \frac{1}{(2i)^2} < \sum_{i=1}^{\infty} \frac{1}{i^2}.
$$