Staff Solutions to Mini-Quiz 10, afternoon

Problem 1 (6 points).
There is a robot that steps between integer positions in 3-dimensional space. Each step of the robot increments one coordinate and leaves the other two unchanged. How many paths can the robot follow going from the origin (1, 1, 1) to (3, 4, 5)?

Solution.

\[
\binom{9}{2, 3, 4}
\]

There is an obvious bijection between such paths and sequences of \(3 - 1 = 2\) x’s, \(4 - 1 = 3\) y’s, and \(5 - 1 = 4\) z’s corresponding to which coordinate gets incremented at each step.

Problem 2 (4 points).
In a standard 52-card deck, what is the smallest \(k\) such that every size \(k\) subset of the 52 cards contains a flush? (flush is defined as 5 cards of the same suit). Briefly explain your answer.

Solution. By the Pigeonhole Principle, there are 4 suits (holes), so \((4)(5 - 1) + 1 = 17\) cards (pigeons) will guarantee that at least 5 cards have the same suit.