Staff Solutions to Mini-Quiz 1, morning

Problem 1 (10 points).
Prove that \( \log_4 6 \) is irrational.

Solution. Suppose to the contrary that \( \log_4 6 \) is rational, so \( \log_4 6 = m/n \) for some positive integers \( m \) and \( n \). So \( m = n \log_4 6 \). Now raising 4 to each side of this equation gives

\[
4^m = 4^{n \log_4 6} = (4^{\log_4 6})^n = 6^n. \tag{1}
\]

But this is impossible, since right hand side of (1) is divisible by 3 and the left hand side is not. This contradiction implies that \( \log_4 6 \) must be irrational.